A STOCHASTIC MODEL OF FLOW-DURATION CURVES FOR RIVERS IN BRITISH COLUMBIA, CANADA

By

DAVID WALTER LEBOUTILLIER

UNIVERSITY OF FLORIDA LIBRARIES

A DISSERTATION PRESENTED TO THE GRADUATE SCHOOL
OF THE UNIVERSITY OF FLORIDA IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

UNIVERSITY OF FLORIDA
1990

ACKNOWLEDGEMENTS

I would like to thank Dr. Peter Waylen for his encouragement, support and guidance through this degree. Dr Waylen's generosity and friendship have provided me with a role model as an academic that I can only hope to follow. The members of my supervisory committee have been invaluable in completing this degree, and their time and help are much appreciated. A special note of thanks must go to Dr. James Henry who always found the way and time to fund my computer account through all these years, and Jan Coyne who helped with all the graphics.

I would like to thank my parents for their letters and phone calls of encouragement and love at all the right times, while I have been working on this degree. My parents have been very supportive through the years of my education, and have always been understanding and patient.

Postgraduate scholarships from the Natural Science and Engineering Research Council of Canada allowed me to begin this work, and without them I would not have completed it. This research was supported by a Doctoral Dissertation Improvement Grant from the National Science Foundation (Grant No. 160945712).

TABLE OF CONTENTS

ACKNOWLEDGEMENTS ii

LIST OF TABLES vi

LIST OF FIGURES vii

ABSTRACT ix

CHAPTERS

1 INTRODUCTION 1

Flow-Duration Curves 1
Use of Flow-Duration Curves 2
Justification and Objectives 2
Quantile Estimation 3
Regionalization 4
Objectives 4
Structure of Presentation 5

2 LITERATURE REVIEW 6

Flow-Duration Curves 6
Construction Techniques 6
Total-Period Technique 7
Complete-Record Technique 7
Average Annual Hydrograph Technique 8
Calendar-Year Technique 8
Discussion 9
Relation to Traditional Frequency Measures 12
Spatial Estimation in Hydrology 16
Basin Scale 16
Regionalization, Prediction and Estimation 18

3 STUDY AREA AND DATA 22

Study Area 22
Rationale 22
General 25
Physiography 26
Precipitation 27
Temperature 33
Streamflow 36
Data 40

4 STATISTICAL MODEL 41

Approach and Theory 41
Order Statistics 41
Parent Distribution 44
Lognormal Distributions 44
Gamma Distributions 46
Generalized Extreme Value Distribution 47
Lognormal Mixture Distribution 48
Spatial Analysis 50
Regionalization 50
Spatial Estimation 51
Concluding Remarks 54

5 RESULTS 56

Statistical Model 58
Regionalization and Prediction 63
Results 63
Spatial Analysis 71
Summary 77
Predictions 77
Conclusions 82

6 FURTHER IMPLICATIONS 84

Extensions of the Basic Model 84
Distributional Results 94
Asymptotic Theory of Extreme Order Statistics 94
Asymptotic Theory of Central and Intermediate Order Statistics 96
Concluding Remarks 102

7 CONCLUSIONS 107

APPENDICES

A LISTING OF GAGING STATIONS 111

B PARAMETER ESTIMATES FOR PARENT DISTRIBUTION 120

C LISTING OF TEST GAGING STATIONS 125

REFERENCE LIST 126

BIOGRAPHICAL SKETCH 136

LIST OF TABLES

Table		Page
1.	Common Moment Estimators	50
2.	Parameter Estimates for Parent Distributions	60
3.	Mixture Distribution Parameters for Parent Distributions	61
4.	Cluster Summary of Parameters of the Mixture Distribution Parent	74
5.	Map Predictions of Parameters of the Mixture Distribution Parent	81
6.	Coefficients of the Regression Correction	89
7.	Normalizing Constants for Maxima	98
8.	Domains of Attraction by Type	99
9.	Sumas River, Annual Distribution Parameters	103

LIST OF FIGURES

Figure		Page
1.	Flow-Duration Curves.	10
2.	Flow-Duration Curves.	11
3.	Physiographic Regions	23
4.	Hydrometric Network	24
5.	Mean Annual Precipitation	28
6.	Sample Climate Stations	30
7.	Northern Climate Stations Mean Monthly Precipitation	31
8.	Southern Climate Stations Mean Monthly Precipitation	32
9.	Northern Climate Stations Monthly Temperatures	34
10.	Southern Climate Stations Monthly Temperatures	35
11.	Sample Gaging Stations	37
12.	Northern Gaging Stations Mean Monthly Discharge	38
13.	Southern Gaging Stations Mean Monthly Discharge	39
14.	Sample Grid	52
15.	Study Rivers, Gage Locations and Average Annual Hydrographs	57
16.	Study Rivers, Parent Frequency Distributions	59
17.	Study Rivers, Observed and Model Flow-Duration Curves	62
18.	Hydrometric Network	64

19. Mean of the First Component 65
20. Standard Deviation of the First Component 66
21. Mixing Proportion 68
22. Mean of the Second Component 69
23. Standard Deviation of the Second Component 70
24. Cluster Membership and Regions 72
25. Average Flow-Duration Curves by Cluster Membership 73
26. Test Rivers, Gage Locations and Average Annual Hydrographs 78
27. Test Rivers, Observed and Predicted Model Flow-Duration Curves ... 79
28. Average Annual Hydrographs 86
29. Sumas River, Means by Rank 87
30. Quesnel River, Means by Rank 88
31. Sumas River, Variances by Rank 90
32. Quesnel River, Variances by Rank 92
33. Sumas River, Annual Distributions 101
34. Sumas River, Flow-Duration Curve with Standard deviations 104

Abstract of Dissertation Presented to the Graduate School
of the University of Florida in Partial Fulfillment of the
Requirements for the Degree Doctor of Philosophy

A STOCHASTIC MODEL OF FLOW-DURATION CURVES FOR RIVERS IN BRITISH COLUMBIA, CANADA

By

David Walter LeBoutillier

August, 1990

Chairperson: Peter Waylen
Major Department: Geography

The estimation of flow quantiles and the regionalization of hydrologic characteristics are two major foci of current hydrologic research. Probabilistic representation of a few individual quantiles, notably the annual flood and low-flow, have been devised, and recently a few of the larger quantiles have been modelled. This research describes and demonstrates the use of a probability model for all flow quantiles in a river.

A flow-duration curve represents the annual flow-frequency characteristics of rivers by depicting the cumulative frequencies for average ranked flows in a river. Generally the process requires the empirical estimations of the mean flow at each of three hundred and sixty five ranks. A model requiring only five parameters is developed by combining the principles of order statistics and traditional flow-

frequency analyses and is applied to flow-duration curves for rivers in the Province of British Columbia, Canada.

Results from the model may be interpreted both statistically and physically and allow the identification of hydrologically similar regions. Streamflows in British Columbia are generated from a number of distinct physical processes operating in highly variable environments. Two of the primary generating processes active in British Columbia are rainfall and spring snowmelt, each of which generates differing hydrograph responses from a given basin. These processes are not exclusive in when and where they operate and combine with other processes, such as summer glacier melt, to create highly complex flow frequency regimes.

The mixing of the streamflow generating processes requires that the statistical model be capable of encompassing the different flow-frequency regimes. Identification of hydrologic regions also depends on the inclusion of these generating processes. The model presented incorporates the physical generating processes of streamflow in both the statistical representation of flow-duration curves and their interpretation. Similarly, the spatial model presents hydrologic regions which correspond to the known physical environment.

Estimation of spatially continuous, hydrologic variables over a region permits the prediction of the parameters of the model for flow-duration curves at ungaged locations. A model for the estimation and generalization of flow-duration curves for rivers in British Columbia, Canada, is developed and tested.

CHAPTER 1
INTRODUCTION

Hydrologic research has investigated several aspects of streamflow by examining just a small part of the flow-frequency regime for a river. Concentration on the annual flood or annual low-flow, which are just segments of the complete regime, is well established. Current research in hydrology is beginning to examine more of the flow-frequency regime by including additional quantiles (Smith, 1989). The entire flow-frequency regime has traditionally been summarized by a flow-duration curve, which describes the average characteristics of all quantiles in a year.

Digitized by the Internet Archive
in 2010 with funding from
University of Florida, George A. Smathers Libraries with support from Lyrasis and the Sloan Foundation

Flow-Duration Curves

A flow-duration curve represents the proportion of time that a particular discharge in a river is observed to be equalled or exceeded during the period of observation. Discharge, which is a continuous variable, may be averaged over hourly, daily, weekly or monthly time periods. The period of observation may be daily, weekly, monthly, annual or the entire recorded time history of the river.

The flow-duration curve has the measure of discharge on the y-axis, and the proportion of time that discharge was equalled or exceeded on the x-axis. It presents a cumulative frequency curve for the magnitude of flows in any river.

http://www.archive.org/details/stochasticmodelo00lebo

Use of Flow-Duration Curves

Traditional engineering applications of flow-duration curves are evaluation of water supply potentials of a river for reservoir design and hydroelectric power generation (Chow, 1964; Foster, 1934; Searcy, 1959). Many water resources planning and design activities at some point consider the flow-frequency behavior of a river--either the highest flows, the lowest flows, the intermediate flows or the average characteristics. Flow-duration curves also provide a measure of the magnitude and frequency of stream power, and hence the erosive capacity of the stream and its ability to generate hydroelectric power (Foster, 1934; Leopold et al., 1964; Richards, 1982).

Most commonly, operational hydrology has been concerned with the risks associated with the largest flow in a year (annual flood), and more recently, the smallest flows. However, little interest has been shown in the use of flow-duration curves to evaluate these and other flow frequencies principally because of the lack of a theoretical basis for their study.

Justification and Objectives

Potter (1987) identifies two major areas of research in operational hydrology: 1) statistical estimation of flood quantiles, and 2) regionalization.

Quantile Estimation

In the last ten years flood frequency analysis has concentrated on the choice of probability distributions to model the annual flood. Commonly used distributions include the Log Pearson Type III (Water Resources Council, 1967), the generalized extreme value distribution (Jenkinson, 1954 & 1969), the Wakeby distribution (Houghton, 1978), the three extreme values types (Potter, 1987) and the lognormal (Chow, 1954). The overriding criterion for success in each study has been the ability to model annual flood frequencies using the best-fit distribution (Cunnane, 1985). Where annual flood frequency behavior has become too complex for a single distribution to model, additional processes have been postulated, and mixture models developed (Rossi et al., 1984; Waylen & Woo, 1982). Despite the success of these models, there remains a need for new approaches and ideas towards the scientific understanding of flood frequency, which include both strong theoretical and physical bases in addition to goodness-of-fit (Klemeš, 1986; Potter, 1987).

This research examines all daily flows generated by a river within a year, a population from which annual floods and low-flows are only special cases which can be incorporated into a more general model. The annual flood by definition is the largest flow observed in a year; similarly all the other flows within a year may be ranked and the probabilities of the discharges at each rank investigated. The flows in a river are considered to reflect the basin inputs and the processes active within the basin. Their probabilities should therefore reflect changing hydrologic environments.

Regionalization

The identification of hydrologic regions is intrinsically geographic and has practical application in the estimation of characteristics for ungaged basins. Recent research has specifically focussed on the investigation and identification of homogenous regions of flow generating processes and basin behavior (Wiltshire, 1986a). Techniques employed include complex regression models (Leith, 1975), index flood approaches (Dalrymple, 1960; Natural Environment Research Council, 1975) and mapping the parameters of probability distributions (Matalas et al., 1975). The identification of homogenous regions requires the arbitrary location of boundaries on to what are essentially continuous and scale-dependent sets of variables. This work acknowledges the spatial heterogeneity of hydrologic processes and attempts to model flow-frequency indicators along a spatial continuum.

Objectives

This dissertation has three major objectives: 1) to select a flow-duration curve construction technique which corresponds to traditional hydrologic measures of risk, and lends itself to the subsequent objectives; 2) to build a theoretically based quantitative model for the form of flow-duration curves which summarizes the probability that a discharge will be equalled or exceeded for a certain proportion of the year; and 3) to develop a spatial technique by which parameters may be

estimated, and the distribution of ranked discharges derived, for any location in the study area.

Structure of Presentation

Chapter 2 presents a review of the literature and builds a framework within which to place this dissertation in the broader perspective of hydrologic research. A description of the study area and data is provided in chapter 3. The theoretical model for flow-duration curves is developed in chapter 4. Chapter 5 presents the results of the model in the context of the hydrology of the study area and develops a set of maps which may be used for spatial prediction. Some additional implications and directions of future research are presented in chapter 6. Chapter 7 presents a summary of this dissertation and some conclusions.

CHAPTER 2
LITERATURE REVIEW

This chapter reviews the literature on the flow-duration curves and places their use within the broader framework of flow-frequency investigation in hydrology. Additionally, the problems of spatial analysis and prediction in both geographic and hydrologic terms are discussed.

Flow-Duration Curves

All discussion in this dissertation is restricted to the most common observation of daily discharge. However, the concepts may be applied directly to other discharge measures such as mean monthly or mean annual flows. Daily discharge at a gage represents the spatially integrated outflow from a basin. The streamflow time series reflects the inputs and storage characteristics of the basin controlled by basin of factors such as climate, topography and geology (Searcy, 1959). The flow-duration curve provides a summary of these factors and the magnitude-frequency relationship for discharge in a basin (Wolman & Miller, 1960).

Construction Techniques

There are four techniques used to construct flow-duration curves. Each approach provides curves which are slightly different in shape and interpretation.

They may be divided into two types: those dealing with the entire record of the river and those providing an annual characterization of the river. Both may be calculated over a water year, however further discussion is restricted to the calendar-year.

Total-Period Technique

All data are sorted in descending order of magnitude. Each observation of discharge becomes the ordinate, and a plotting position, such as the Weibull ($p_j = j/(n+1)$ where j is the rank), is the abscissa. The result is that each observation is plotted as one segment of the curve, (Foster, 1934; Searcy, 1959; Watson, 1934).

Foster (1934) describes this technique as the most accurate, as it faithfully represents the entire flow-frequency time history of the river. Construction is time intensive, and inclusion of additional data requires re-drafting of the entire curve. Important annual observations, such as the annual flood, are lost because all historic data are aggregated before deriving the curve. This curve should be interpreted as showing the proportion of time in the entire record that a particular discharge is equalled or exceeded.

Complete-Record Technique

Observed data are assigned to a set of equally-sized classes. The curve is generated by graphing the class midpoint (y-axis) against cumulative relative frequency (x-axis) and represents the entire flow-frequency history of the river, but has the concomitant advantages and disadvantages of classified data. Searcy (1959) describes this as a more accurate form of the curve when compared to the calendar-year curve. New observations are easily added to this curve, and updating requires

only the redrawing of each class. Annual information is lost in constructing this curve and care must be taken in extracting values from the classified data.

Average Annual Hydrograph Technique

The average annual hydrograph is constructed by calculating mean discharges by day, over the years of record. The resulting average annual hydrograph is then sorted in descending order and average daily discharge plotted against rank. The curve thereby retains annual information about the average discharge at each day.

Because averaging is conducted by day, and subsequently sorted, daily variances, which tend to be high, are unrepresented in the finished product. The highest flow in a river does not occur on the same day of every year, nor is every year similar. The result is to mix the individual ranked flows and to lose detail about annual variability. Updating the curve is relatively easy, as only 365 points have to be recalculated per year. The primary benefit of this technique is that years of only partial discharge record may be included. Interpreting this curve is very difficult as the information it presents is the ranked characterization of the seasonality function of the river.

Calendar-Year Technique

Observations of discharge for each year are ranked and averages by rank calculated. The plot of average by rank is the annual flow-duration curve. This curve truly represents the average ranked flow behavior in a river. Traditional flow-frequency measures such as the mean annual flood, are preserved. Searcy (1959)

describes the curve as less accurate than the complete-record, but acknowledges that it is the only true representation of the year-to-year behavior.

Discussion

Each construction technique has particular merits and produces very different results. Both the total-period and the complete-record approaches provide information about the entire recorded time history of the river. The average annual hydrograph and calendar-year curves provide annual information.

Figures 1 (A & B) and 2 (A & B) compare the curves resulting from the four construction techniques for the Sumas River in British Columbia. The total-period and complete-record curves (Figure 1) are very similar. The total-period curve tends to give slightly higher discharges for all percentages than does the complete-record. This is especially noticeable at both the lowest and highest percentage regions of the curve. The greatest handicap of the complete-record technique is the use of classes. Any year of record may not contribute to all classes evenly, class sizes will vary greatly and the resulting curve may be very different for different numbers of classes.

In Figure 2 both axes are identical and the graphs may be accurately compared, yet the average annual hydrograph and calendar-year curves are very different. The average annual hydrograph technique gives lower discharge values at the low-rank end of the curve than does the calendar-year curve, while the middle regions of both curves are very similar. For low ranks, the average annual

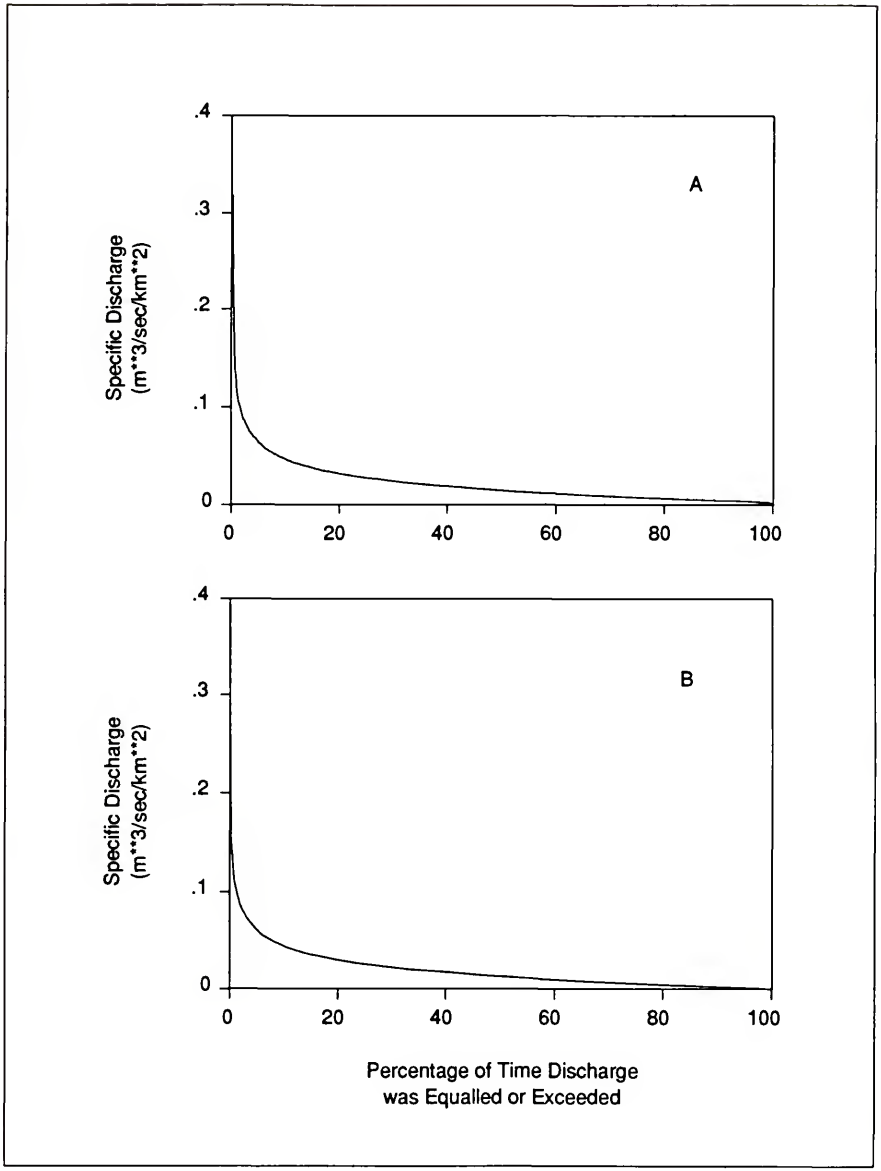

Figure 1. Flow-Duration Curves. A) Total-Period; B) Complete-Record

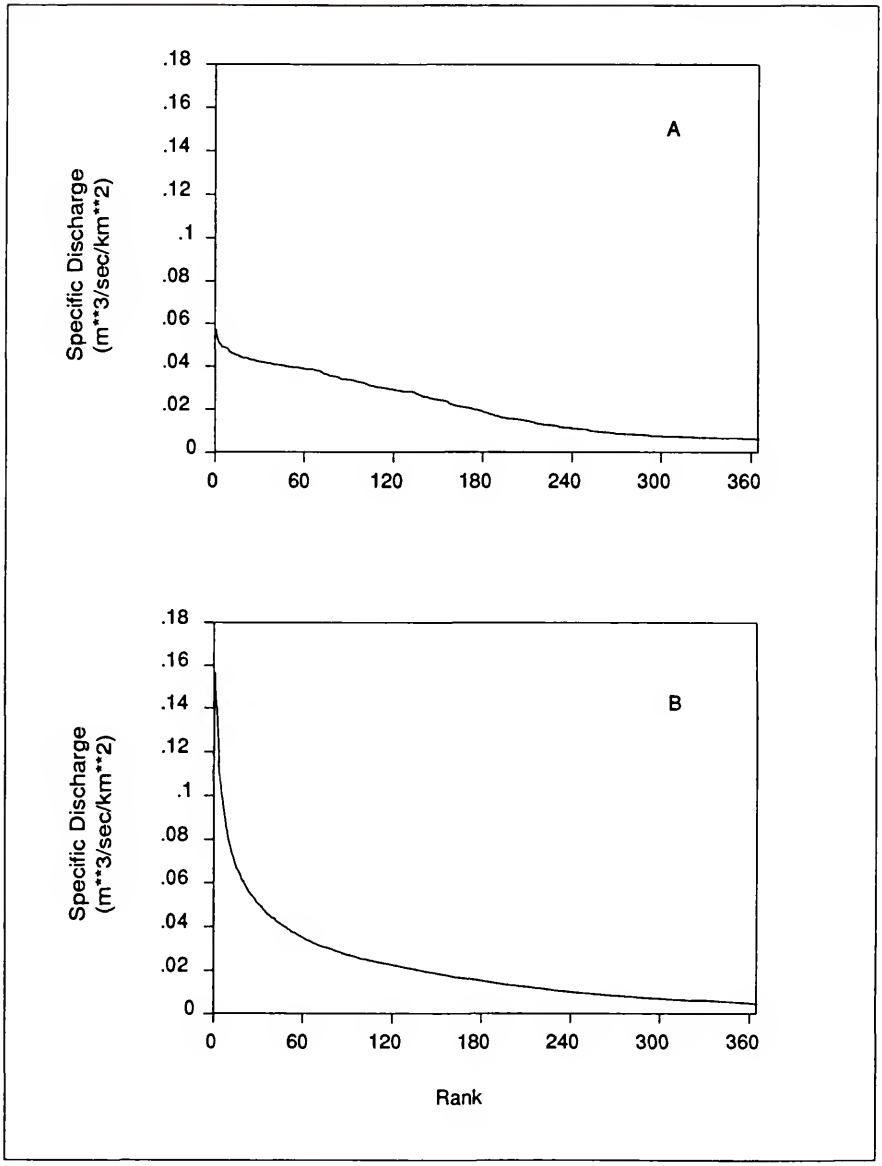

Figure 2. Flow-Duration Curves. A) Average Annual Hydrograph; B) Calendar-Year

hydrograph curve provides estimates of discharge which are one third of those provided by the calendar-year curve.

Comparing the curves across techniques, the complete-record curve (Figure 1-B), recommended by the U.S.G.S. (Searcy, 1959), tends to estimate higher discharges for rare events than does the calendar-year curve. For most of the values on the x-axis, the curves are identical, diverging again at the low discharge end of the curve where the complete-record technique gives values of discharge less than the calendar-year.

Flow-duration curves are generally used to make qualitative comparisons between different flow regimes and to infer basin characteristics. Two of the four curves have no theoretical parallel in statistical theory. There is however a similarity between the construction of both the calendar-year and total-period curves and the field of order statistics.

Relation to Traditional Frequency Measures

One of the most significant factors in the emergence of hydrology as a separate and distinct science is the full-fledged use of probabilistic and random process techniques in the study of hydrologic problems (Bras & Rodriguez-Iturbe, 1985). The probabilistic approach has allowed hydrologists to model phenomena which would have been impossible or too expensive to model deterministically, phenomena as diverse as hydraulic conductivity (Freeze, 1975) and paleoflood events (Baker, 1985). The ability of stochastic models to represent what is known and

important about the phenomenon and to represent the unknown or unobserved portions by probability distributions, has freed hydrologists from over-abstraction or 'mathematistry' (Box, 1976; Klemeš, 1986). This permits the marrying of a strong theoretical basis to a sound physical basis in the models (Potter, 1987).

Current hydrologic frequency analysis emphasizes a better understanding of the physical processes affecting the flood frequency characteristics in order to construct improved risk prediction models (Klemeš, 1986; Potter, 1987). Knowledge of hydrologic generating processes has led to increased emphasis on the role of the drainage basin, its inputs via the hydrologic cycle and their interaction with the basin (Dooge, 1986). The criteria for an improved hydrologic model are now not only goodness-of-fit but also physical reasonableness and relative parsimony (Yevjevich & Obeysekera, 1985). Such models show an increasing analytical sophistication (Potter, 1987), and may include mixed processes (Rossi et al., 1984; Waylen, 1985a), time series approaches (Smith & Karr, 1986) and the analysis of regionalized variables (Acreman, 1985).

Analysis of annual floods is perhaps the most intensely, widely and longest studied probabilistic problem in hydrology. This concentration is largely historical and is continued out of convenience and the fact that there are few better measures of extraordinary flooding than the annual flood. Approaches to the problem have ranged from strictly empirical modelling (Hazen 1930; Jarvis, 1936), through the application of extreme value theory (Fisher & Tippett, 1928; Gumbel 1941 & 1958),

and lately to the selection of best-fit distributions (Lamberti & Pilati, 1985) and exact analytical solutions from other forms of analysis (Zelenhasic, 1970).

Annual flood studies can be divided into two broad approaches, one stressing extreme value theory as a justification of the choice of a particular distribution, the other abandoning theoretical bases and stressing goodness-of-fit. Extreme value distributions have been rejected on the grounds that the physical generating processes of streamflow do not meet the mathematical assumptions of the theory (Gupta et al., 1976). However, some argument for the use of extreme value theory is still made, on the grounds that the annual flood is the extreme value of discharge observed in a year (Cunnane, 1985).

Extreme value theory yields the three extreme value (EV) distributions, the EV 1 or Gumbel distribution, the EV 2 or Frechet distribution, and the EV 3 or Weibull distribution. Jenkinson's (1954 & 1969) proposal that a generalized extreme value distribution (GEV) be used, has been widely adopted (Hosking & Wallis, 1986). The extreme value approach has been successful (Waylen & Woo, 1983; Wiltshire, 1986b), despite its apparent lack of theoretical support and the problems of small sample sizes, independence of floods and non-stationarity in the distributions of their magnitudes (Fiorentino et al., 1985).

Conversely, the disregard for any theoretical basis in the selection of a probability distribution to model annual floods has resulted in a wide variety of distributions, each with its own set of advocates. Distributions chosen have included the EV I or Gumbel (Waylen & Woo, 1982 & 1983), EV II and EV III extreme

value distributions, a generalized extreme value distribution (Wiltshire, 1986b), mixtures of the extreme value type I (Waylen & Caviedes, 1986; Waylen & Woo, 1982), the Pearson type III (Matalas & Wallis, 1973) and log Pearson type III (Water Resources Council, 1967), the lognormal (Chow, 1954), the gamma distributions and mixtures of the analytical form from the partial duration series (Rossi et al., 1984). These distributional choices have no theoretical justification, but they have the ability to fit the data at a station and permit spatial comparison. Regional flood models have also been proposed that require the assumption of dependence structures between sites and the parameters of flood distributions (National Research Council, 1988).

The analysis of annual low-flows has always been the less studied of the annual measures. Numerical advances parallel those of annual flood research. Gumbel (1954a) applied extreme value theory to the study of droughts, as reflected by the lowest discharge in a year. He, and later, Matalas (1963), used the EV III extreme value distribution, which is bounded in the direction of interest. Waylen and Woo (1987) have successfully used mixtures of extreme value distributions to represent the differing low-flow generating processes in snowmelt rivers. Often the annual low-flow is used as a surrogate for measuring hydrologic drought. Dracup et al. (1980b) differentiate between low-flows and droughts on the basis of streamflow and the period over which the low-flow is estimated. Chang and Boyer (1977) estimate the annual 7-day average low-flow with a 10 year return period from watershed and climatic parameters. Dracup et al. (1980a & b) and Zelenhasic and

Salvi (1987) used the partial duration series to define hydrologic droughts on the basis of streamflow below some level.

Spatial Estimation in Hydrology

When hydrologic records are available they are often short and may contain sampling errors. Simulation or extension of these records simply preserves and may amplify these errors. Frequently records are simply not available for the river of interest but are available for nearby rivers. It is therefore often desirable to spatially aggregate available records, thereby enhancing the regional record and increasing the effective sample size.

Within the greater issue of the spatial problem in hydrologic estimation, three separate questions may be identified: 1) At what scale should an analysis be conducted? 2) Should the data be regionalized, and how should the regions be defined? 3) What technique should be used to make predictions?

Basin Scale

Analysis of hydrologic data, in particular streamflow, is frequently undertaken at the basin scale. Streamflow is an integrated measure of all the processes and influences active within that basin. Increased basin area generally produces greater inputs and increased basin storage. Given the range of possible basin sizes, Klemeš (1983) warns that hydrologists should be sensitive to the differences in behavior

resulting from basin area and to avoid direct comparisons between the large and small.

The effects of the drainage basin are most pronounced in the frequency distribution of daily discharges. Greater basin area promotes a larger mean daily discharge and lower skewness. Daily variance on the other hand appears to be largely a function of seasonality.

In terms of physical effects, the presence of large amounts of lake storage produces a more even distribution of flows and moderates the range of flows. Snow and ice storage produces rapid releases of water during the melt season, and consequently generate the highest sustained flows. Ice storage in the form of glaciers releases water more consistently during the warm season. The role soils, geology and slope of the basin as controls on ground water contributions are noticeable in the low-flow season.

The basic technique for 'standardizing' flows from a variety of drainage basins, of varying sizes, is to divide discharge by drainage area (Smith, 1989), yielding discharge in units of volume per unit time per unit area (specific runoff). This technique allows a crude comparison between large and small basins. Basin area is chosen because it is generally easy to measure and is strongly related to other very important characteristics such as relief, slope, drainage density and basin shape (Gregory & Walling, 1973). All else constant, the magnitude of standardized basin runoff is inversely proportional to catchment area.

Flows may also be standardized by 'indexing' them as ratios of mean annual flow or mean annual flood (Acreman, 1985; Natural Environment Research Council, 1975). Acreman (1985) further suggests that the denominators in the indices may be generated from basin parameters and not from the record itself. A third technique, advocated by Fiorentino et al. (1985) and Rossi et al. (1984), standardizes flows by use of estimated parameters, similar to a z-score. Similar to the index flood approaches, basin-scale data may be expressed as standardized values at region scale. This technique involves aggregation of similar basin's data to get larger sample sizes and more stable and less-biased parameter estimates, which lend themselves to the analysis of annual flood data (Dalrymple, 1960; Natural Environment Research Council, 1975).

Regionalization, Prediction and Estimation

There are two major approaches to the analysis of spatial variation in hydrology: 1) identification of homogenous drainage basins and 2) modelling the variability of hydrologic models continuously over space. In the first approach, basins possessing similar statistical properties are grouped and are examined for similarity and in the second, the parameters estimated in each basin are mapped and their spatial pattern expressed mathematically.

Homogeneity statistics are based on physical and/or flow characteristics of the basin (Wiltshire, 1986a & b). Soil type, average slope, lake storage, snow or ice storage, mean annual precipitation, mean annual temperature and basin area have

all been used as physical indicators (Gottschalk, 1985; Matalas, 1967; Wiltshire, 1985). Wiltshire (1985) employed climatological variables including average annual rainfall in combination with some basin morphology measures as the basis for classification. Flow characteristics, index floods, distribution type, parameter values of a regional parent distribution, estimated discharge for a particular quantile (or return period) or general statistical measures may be put to the same purpose (Matalas et al., 1975; Natural Environment Research Council, 1975; Wiltshire, 1986a).

Inherent in the idea of geographic regionalization is classification, the technique by which a site observation is determined to belong to a particular region. The classification of regions is performed on the basis of two criteria: 1) minimizing the variability within each region or class and 2) maximizing the differences between classes (Smith, 1975).

Hydrology has stressed the aspect of homogeneity, or minimizing the within-class variability, as the basis for determining regions. A variety of statistics have been devised (Lettenmaier & Potter, 1985; Lettenmaier et al., 1987; Wiltshire, 1986a & b), to determine regional homogeneity, the simplest of which is the regional mean coefficient of variation. The presence of a common underlying flood frequency distribution which varies in only a few parameters has been a basic assumption of hydrologic regionalization. Regional models have been shown to be resistant to mis-specification of the underlying distribution (Lettenmaier et al., 1987), regional heterogeneity (Lettenmaier et al., 1987) and intersite dependence (Hosking & Wallis,

1988). However, Lettenmaier et al. (1987, page 313) identify "the lack of a physical basis for determining the form of the underlying flood frequency distribution" as a major complication in the use of regionalization techniques. Assignment of basins to a region is completed through statistical techniques including cluster analysis (Mosley, 1981), discriminant analysis (Waylen & Woo, 1984), analysis of variance (Wiltshire 1985 & 1986a), principal components and pairwise grouping (Gottschalk, 1985).

Continuous mapping techniques such as trend surfaces (Unwin, 1975; Woo & Waylen, 1984), multiple regression (Leith, 1976 & 1978) and kriging (Gambolati & Volpi, 1979; Olea, 1974; Woolhiser & Roldan, 1986) have been used to describe the variability of the parameters of a regional parent flood distribution (Matalas et al., 1975), or mixed flood frequency distributions (Woo & Waylen, 1984). Prediction of characteristics of ungaged locations requires estimated surface coefficients, basin location and the necessary basin variables. The accuracy of the results is determined by the reliability of the estimated coefficients and of the measured basin variables. The technique of kriging is largely a mapping tool and provides much better contour results than most conventional mapping techniques (Olea, 1974). The mapped results from kriging do not provide the kind of false accuracy of the other techniques but, at the same time, kriging does not allow mathematical prediction, only interpolation, from a map.

Geographic regionalization aims for the identification of regions of underlying common behavior and characteristics for the goal of spatial generalization with the

added requirement of spatial continuity (Amedeo & Golledge, 1975; Smith, 1975). Hydrologic regionalization aims for the identification of regions of homogeneity in the desired characteristics for the goal of substituting space for time and lengthening the effective record of hydrologic observations; this does not require geographic proximity (Lettenmaier et al., 1987; Wiltshire 1985).

CHAPTER 3
STUDY AREA AND DATA

This chapter presents a brief overview of the climate and physiography of British Columbia, Canada. The description is aided by a series of maps and graphs to illustrate the wide variety of climatic and streamflow regimes observed over the province. A description of the data and their source is provided in the last section of this chapter.

Study Area

Rationale

The province of British Columbia, Canada (Figure 3) is chosen as the study area because of the following significant advantages:

1) A large number of excellent quality discharge records exist for the area. Many records begin in the 1920s, and more than 100 possess at least fifteen years of complete record (see Appendix A).

2) The density of gaging stations exceed the minimum standard set by the World Meteorological Organization (W.M.O.) for mountainous terrain of one station for each 5,000 km^2. British Columbia has one station for each 4700 km^2 (Kreuder, 1979; World Meteorological Organization, 1981).

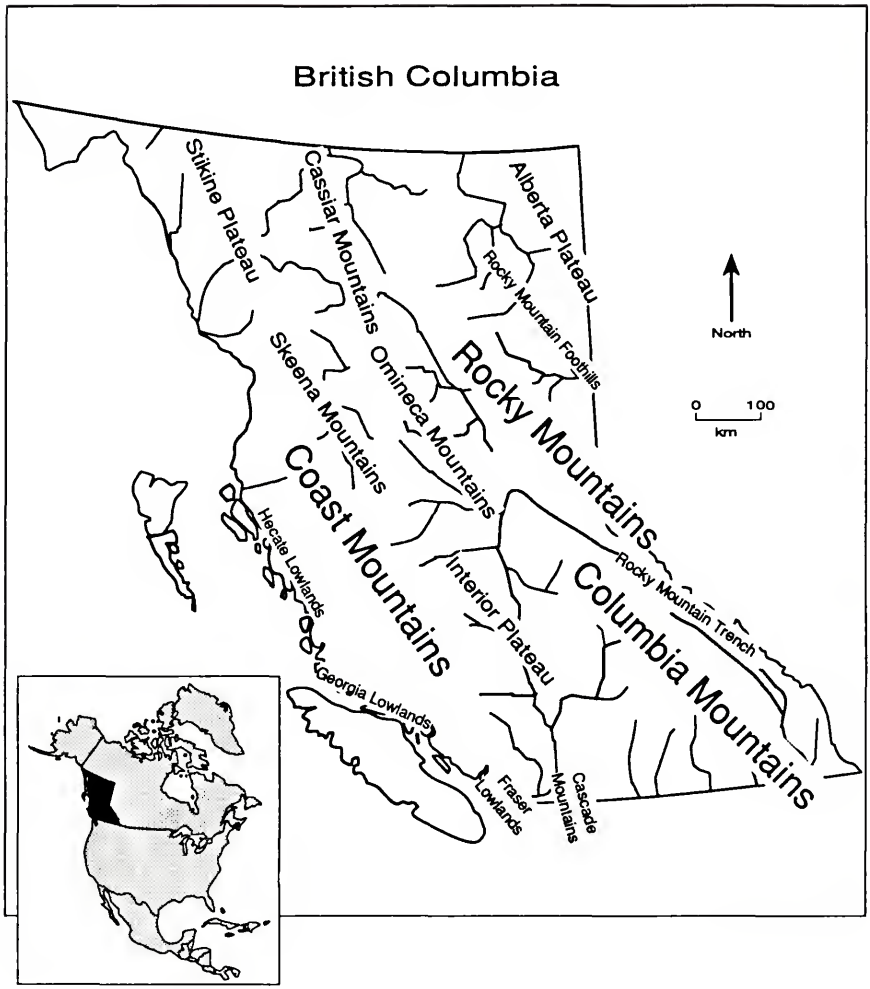

Figure 3. Physiographic Regions

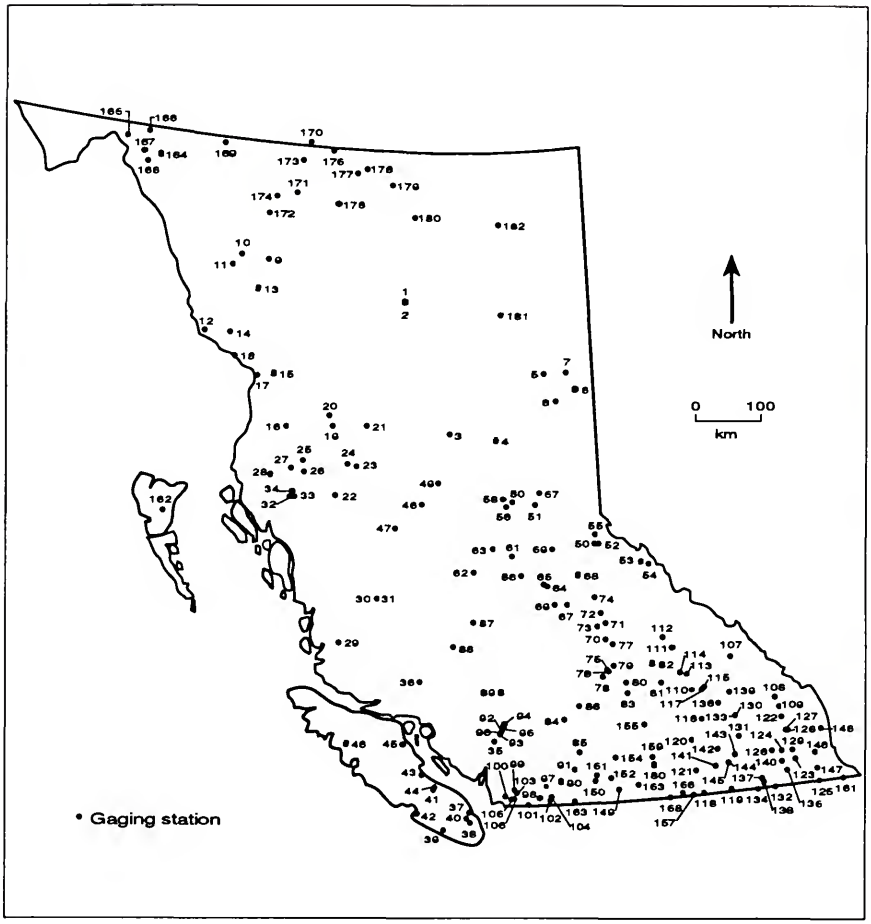

Figure 4. Hydrometric Network

3) The network (Figure 4) of gaging stations has been designed to cover as many of the major rivers as possible (Kreuder, 1979) and to provide a number of transect lines across the province.

4) There are many large and small rivers whose flows are not significantly modified by human activity in a variety of physiographic and hydrologic environments.

On this basis, one hundred and eighty two gaging stations are included for analysis in this research. Figure 4 shows the hydrometric network of these gaging stations, and Appendix A lists each gage, it's full name and location. To be included in the analysis, the gage must be located on a basin without substantial modification such as diversions or dams and possess nineteen or more years of complete record. In areas of poor spatial coverage, gages with more than sixteen years of record are used.

General

The province (Figure 3) occupies an area of nearly 1 million square kilometers bounded to the south by the 49'th parallel and to the north by the 60'th. It's northeastern boundary with the province of Alberta is marked by the meridian, 120°W, and follows the continental divide south and east from a latitude of approximately 53°30'N, meeting the U.S./Canada border at 114°W. To the southwest, the province is bounded by the Pacific Ocean and to the northwest by the panhandle of Alaska. Physiographically most of the province encompasses a series

of diverse regions collectively termed the Western Cordillera, with the exception of the extreme northeast, which occupies the northern interior plains (Department of Energy, Mines and Resources, Canada, 1973). The following brief review of the physical geography of the area will restrict itself to those factors directly related to streamflow.

Physiography

The Western Cordillera is composed of three principle regions, or systems, which run sub-parallel northwest-southeast. The geologic history of the region is documented by Monger and Price (1979). The western system is fringed by areas of coastal plains, the Fraser, Georgia and Hecate Lowlands, where elevations seldom exceed 50m. Immediately abutting these to the east, and frequently intersecting the fiorded coastline, lies a high mountainous region of the Coast and Cascade Mountains, which attain over 3000m of elevation.

The interior system is a complex of mountains and plateaus. The Stikine and Nechako Plateaus lie north and south, respectively, of the Skeena and Omineca Mountains. They comprise subdued relief with a regional surface at about 750m. Occasionally the relief is interrupted by Cenozoic Volcanoes which rise above the surface to about 2000m and by major river valleys, which may be as much as 600m below the plateau surface. Elevations in the intervening mountains are generally between 2000 and 2500m. The eastern portions of the interior system rise from the plateaus to the Columbia Mountains (in the south) and the Cassiar Mountains (in

the north), which ultimately attain elevations comparable to the Coast and Rocky Mountains.

The eastern system of the northern and southern Rocky Mountains is separated from the interior system by an ancient subduction zone, currently forming the low lying Rocky Mountain Trench. Elevations in this system are some of the highest found in Canada, frequently supporting permanent glacial cover. The Trench is presently occupied by the Columbia, Fraser, Parsnip and Finlay Rivers from the U.S. to the Yukon border.

The Peace, Hay and Liard Rivers of the Arctic drainage flow northeastwards towards the MacKenzie River through the Rocky Mountain Foothills and across the Alberta Plateau to the Fort Nelson and Peace River Lowlands, where elevations range between 300 and 600, in a gently undulating landscape.

Precipitation

The following discussion of climatological and hydrological variables are based on statistics compiled by Environment Canada (1982 & 1988).

The two parallel systems of mountains separated by the interior plateau lie across the dominant westerly winds at these latitudes and exert considerable influence on the pattern of mean annual precipitation (Figure 5). On the windward side of the Coast Mountains mean annual precipitation may exceed 3000mm (Farley, 1979) with a marked winter seasonal maxima in an Alpine Maritime Climate (Fraser River Board, 1958). Precipitation declines rapidly on the Interior Plateau to as little

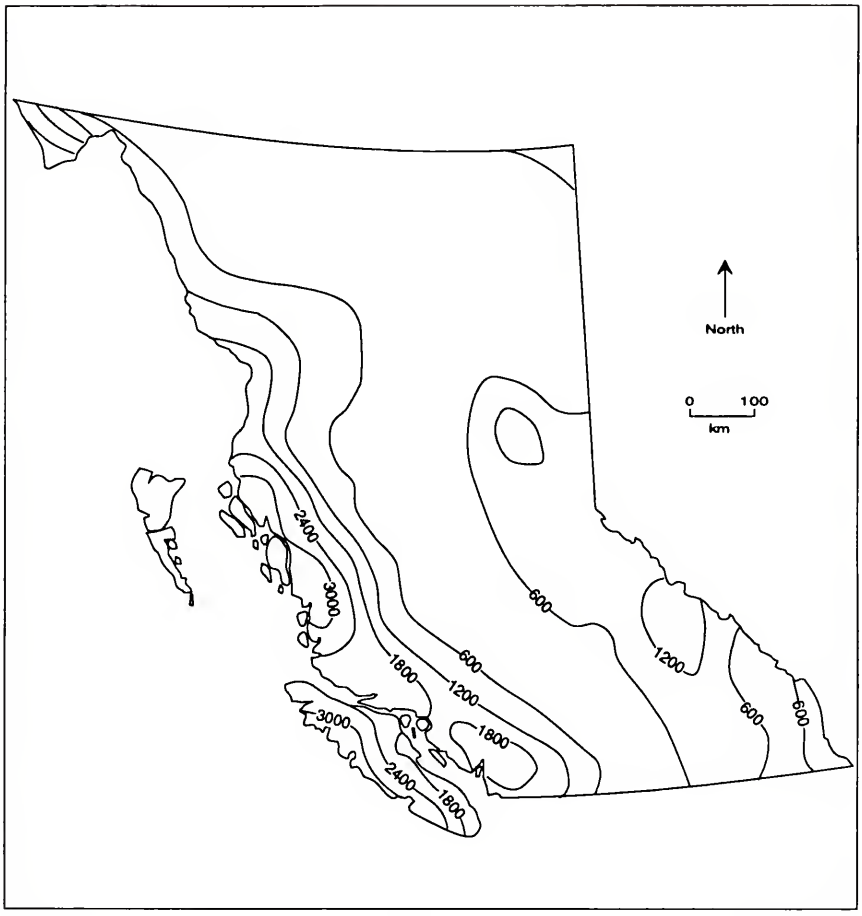

Figure 5. Mean Annual Precipitation (Contour Interval = 600mm)

as 200mm in the southern Dry Continental Climate or 400mm in the northern Humid Continental Climate. Precipitation is generally uniformly distributed throughout the year with the possibility of a slight summer maximum resulting from convective activity.

Increased elevations in the Columbia, Cassiar and Rocky Mountains produce annual precipitation in the order of 1500mm with a winter maximum. This Alpine Humid Continental Climate is highly variable locally; the Rocky Mountain Trench lies in a very pronounced region of rainshadow. To the north of the Skeena and Omineca Mountains the altitude of the Rocky Mountains diminishes and the pattern of rainfall becomes increasingly controlled by latitude. All but the most westerly extremes of the northern part of the province experience 300 and 500mm of annual precipitation and a summer maximum in a climate described as Alpine Subarctic.

Figure 6 locates twelve sample climate stations which demonstrate the mean monthly precipitation regimes of the Province (Figures 7 and 8). Qualicum River and Chilliwack are located along the Coast Mountains and experience a winter rainfall maximum, which progresses towards a fall maximum, at Kitimat. Bralorne, Telegraph Creek and Fort Babine are located in the rainshadow of the Coast Mountains; they show an even distribution of precipitation through the year, with a slight increase of winter precipitation in the south at Bralorne. Smith River, Pink Mountain, Puntchesakut Lake and Hemp Creek have distinct summer maxima in precipitation. Vernon and Cranbrook show a mixing of both winter and summer maxima leading to an almost uniform distribution of precipitation through the year.

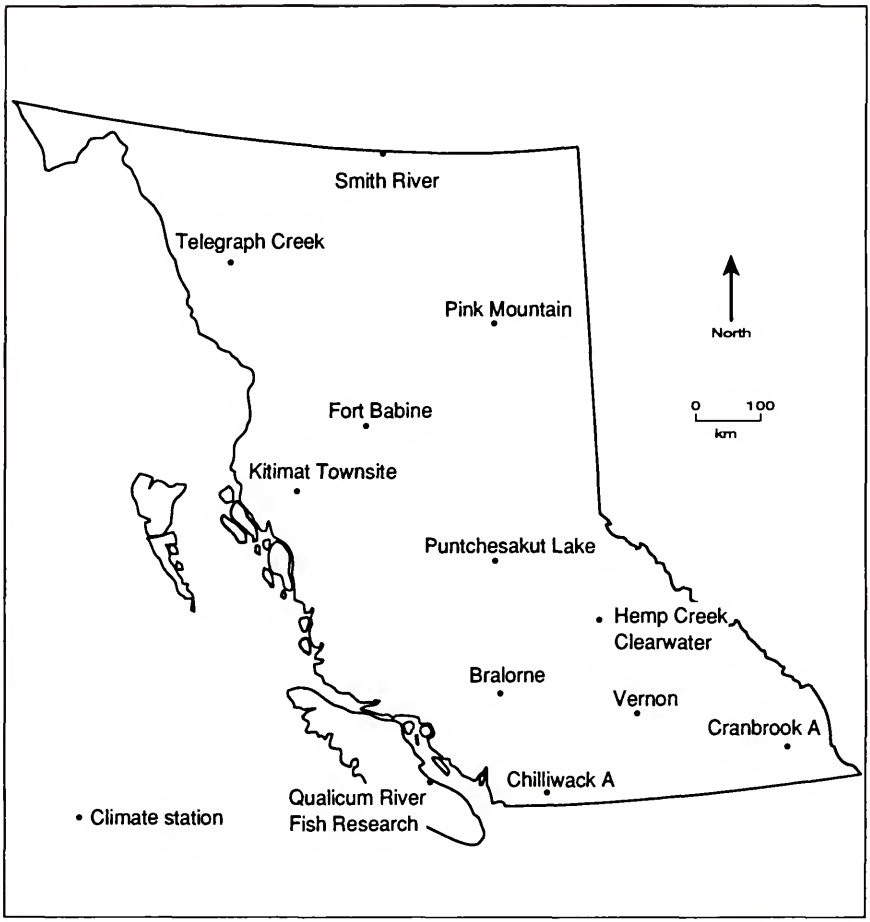

Figure 6. Sample Climate Stations

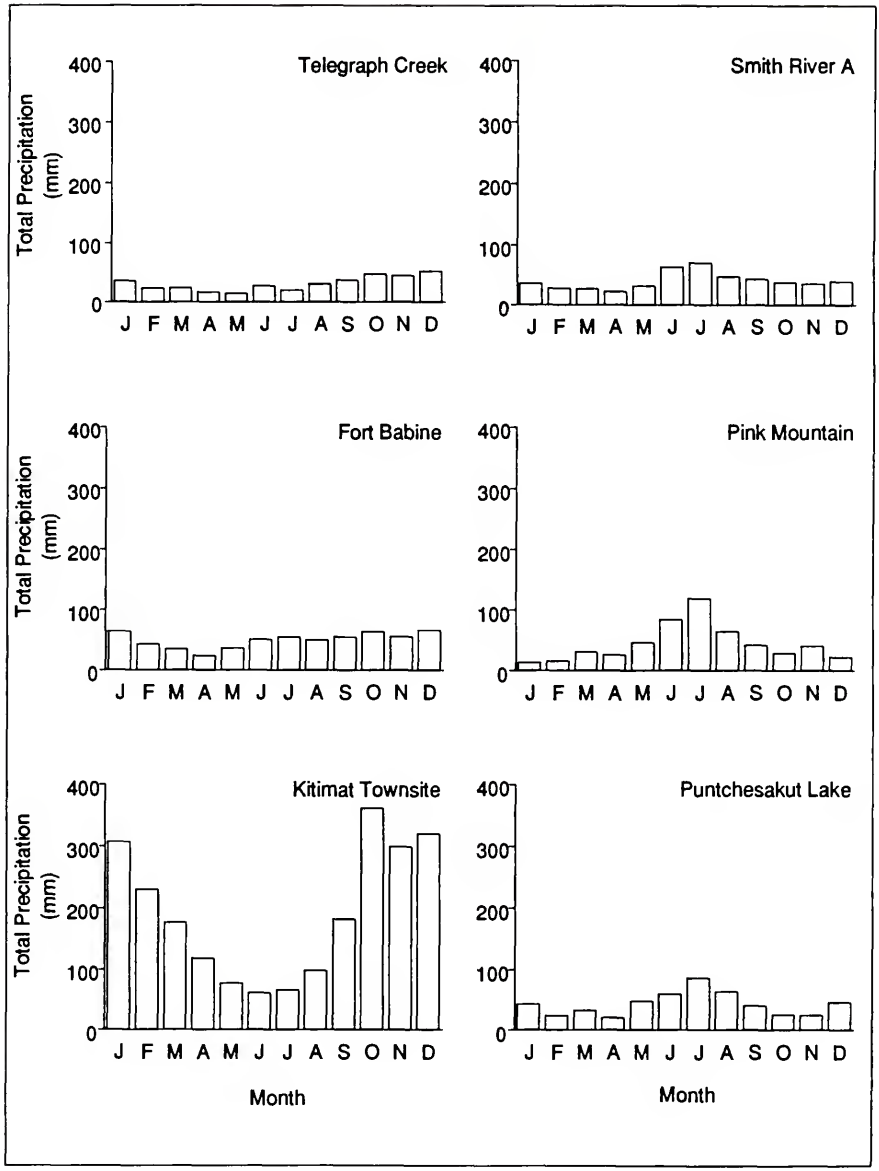

Figure 7. Northern Climate Stations Mean Monthly Precipitation

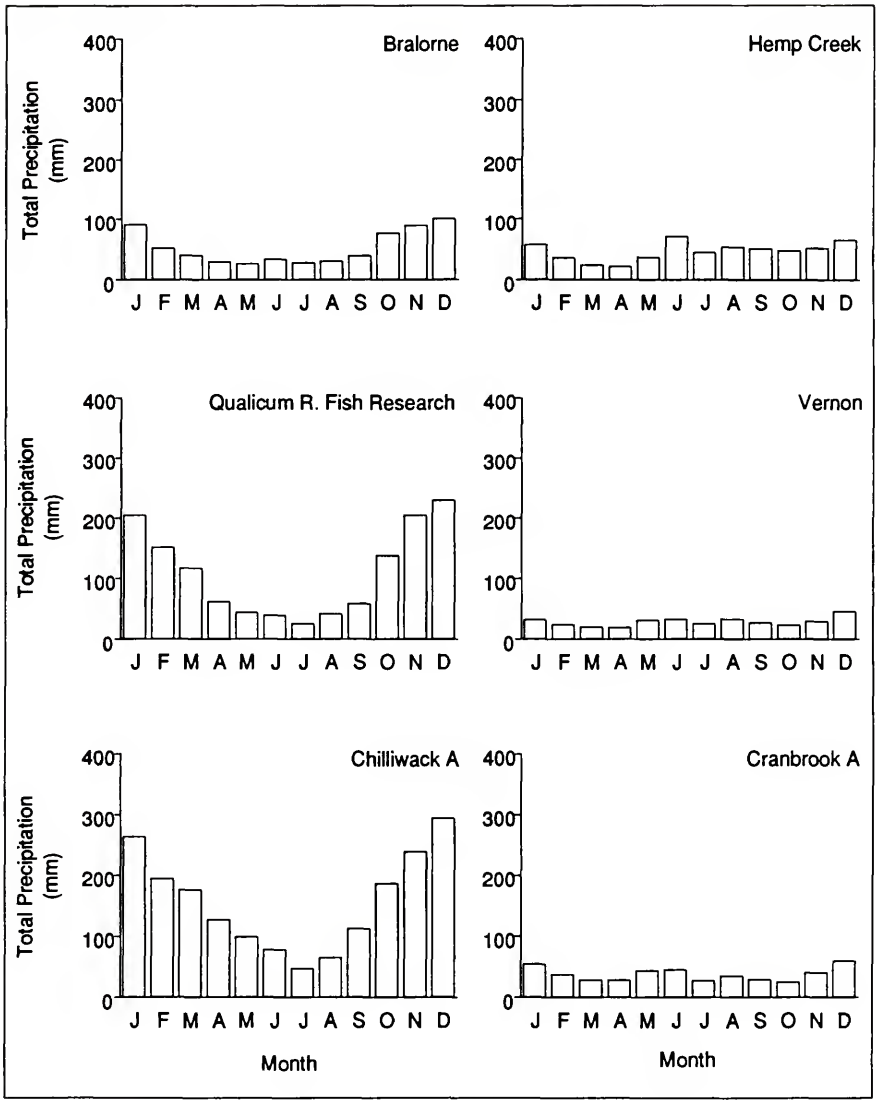

Figure 8. Southern Climate Stations Mean Monthly Precipitation

Temperature

Monthly temperatures (Figures 9 and 10) are primarily a function of elevation, latitude and distance from the Pacific Ocean. Continentality increases rapidly inland as the Coast Mountains effectively exclude the moderating effect of the Pacific. Application of a simple adiabatic lapse rate to the 3000m of available relief gives some idea of the influence of elevation. Mean annual temperatures range from above 10°C on the Pacific southwest to well below zero in the Northern Interior. The most significant effect that temperature has on streamflow is in determining the timing and quantities of water released from snow and ice storage (Pipes et al., 1970). On the Fraser Lowland snow makes up less than two percent of total annual precipitation while it may constitute greater than 70 percent in the northeast mountains.

In general, the lowest monthly temperature occurs in January and the highest in July and August. The moderating influence of the Pacific Ocean is shown in the thermographs of Chilliwack and Qualicum River where average temperatures remain above 0°C through the year. Both eastward and northward the thermal regime is marked by increasing annual range and longer, colder winters. The influence of elevation is not readily demonstrated in these thermographs, as most climate stations in British Columbia are found in the valley bottoms with elevations less than 1,000m.

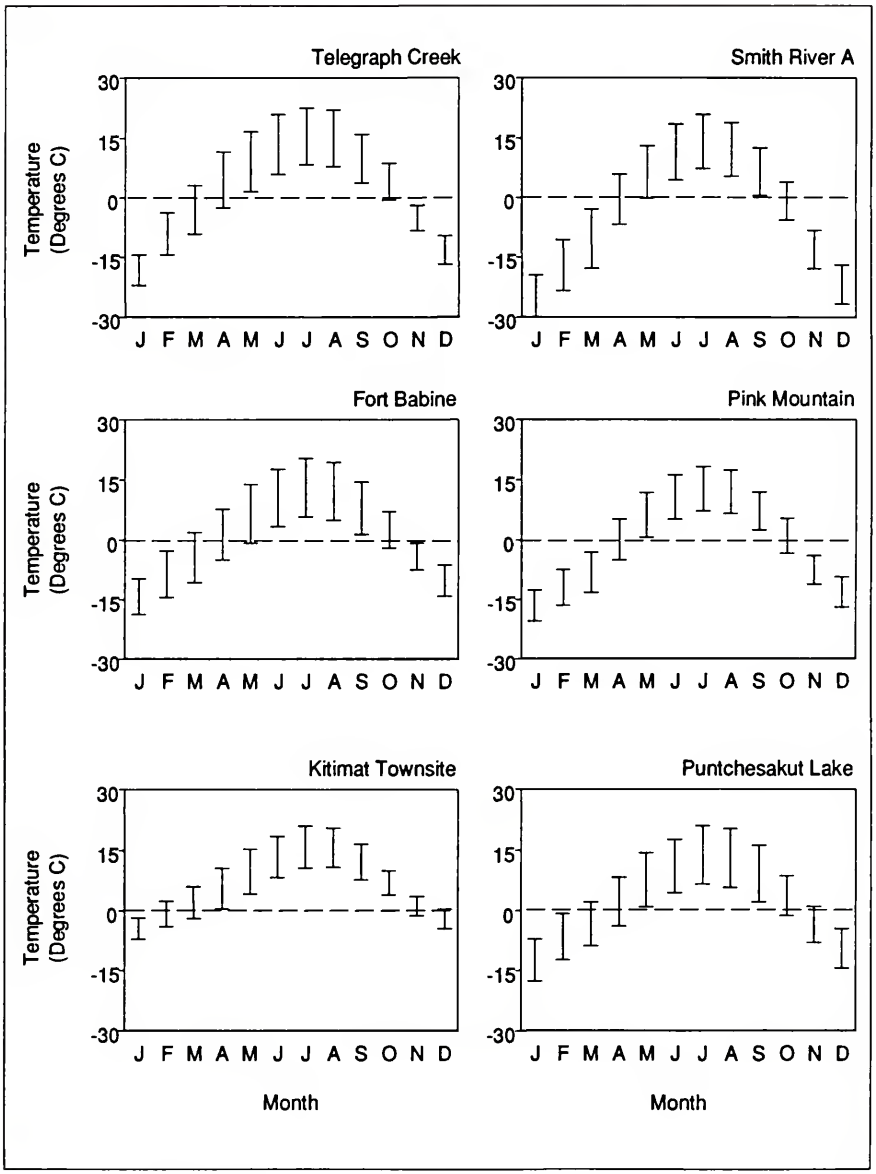

Figure 9. Northern Climate Stations Monthly Temperatures

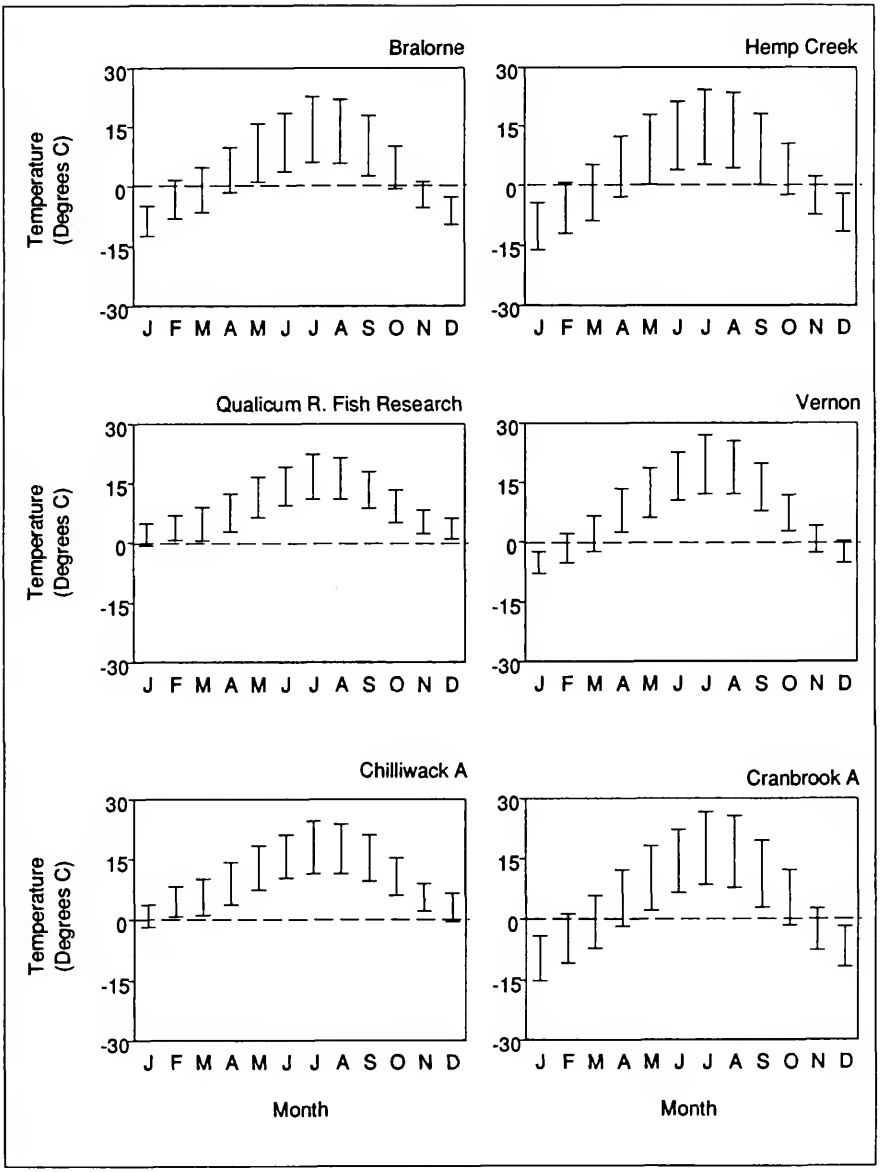

Figure 10. Southern Climate Stations Monthly Temperatures

Streamflow

All streamflow regimes in the study area are highly seasonal. With the exception of Vancouver Island and the west side of the Coast Mountains, most are dominated by snowmelt and have pronounced winter minima. At higher elevations and latitudes glacial melt becomes an important factor in maintaining summer high flows. Along the coast the intrusion of warm moist Pacific air and accompanying frontal systems may produce heavy winter rains, though their influence diminishes inland.

Figure 11 shows twelve gaging stations on rivers located close to the previous twelve climate stations. The different monthly flow regimes across the province are depicted in Figures 12 and 13. Little Qualicum River on Vancouver Island experiences a winter maximum. This basic pattern is modified in the west-facing slopes of the Coast and Cascade Mountains to include spring snowmelt and summer glacier melt at the higher elevations (Slesse Creek and Kitimat River). The interior of the province is dominated by snowmelt regimes (Coldstream and Kootenay Rivers) of much smaller magnitude than the coastal basins and has no winter component. The magnitude of the summer discharge peak largely reflects the amount of snowfall and snow storage in each basin. Gages are located in the valleys but the gaged flows represent the runoff generated from relatively large areas frequently well above the gage itself.

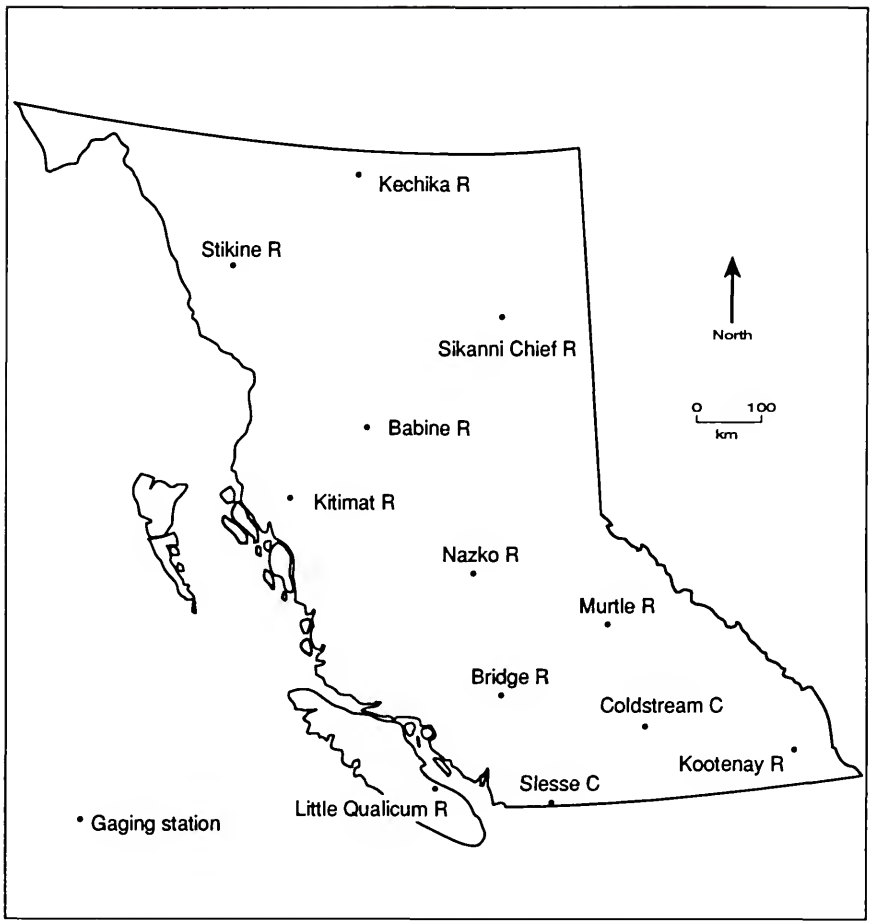

Figure 11. Sample Gaging Stations

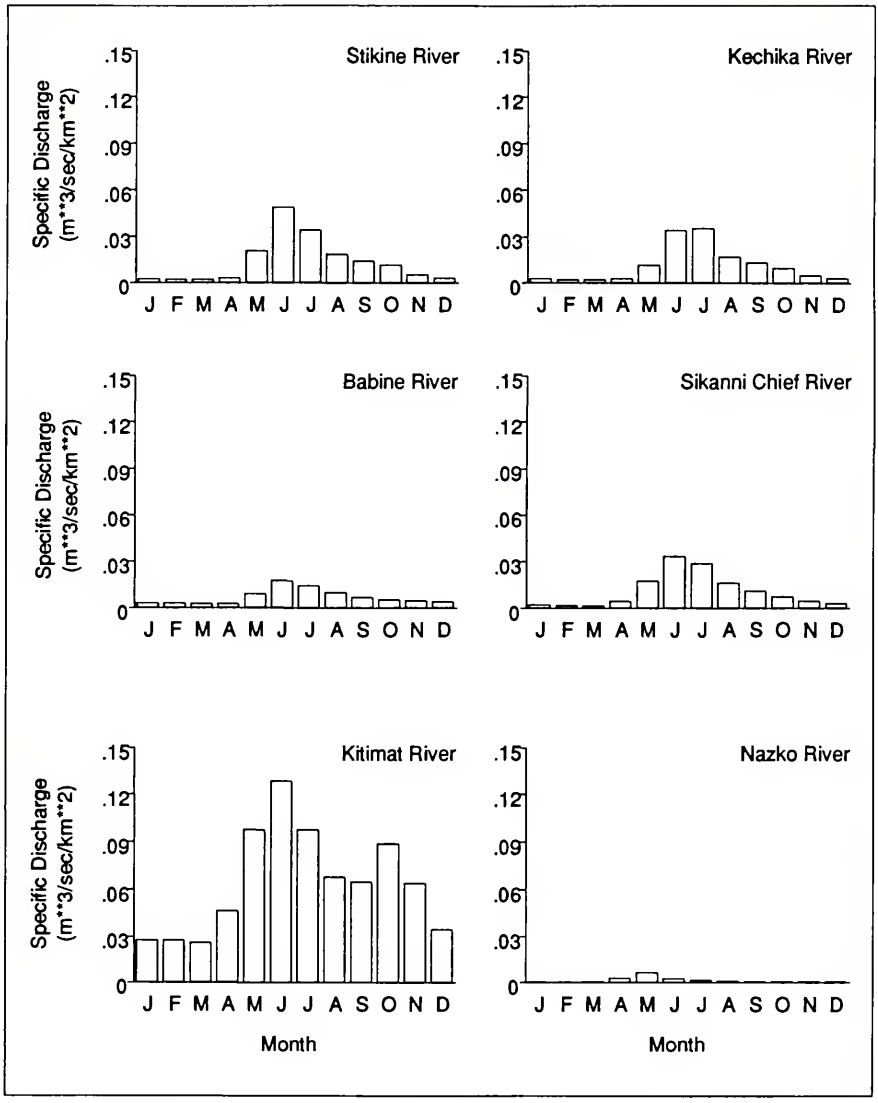

Figure 12. Northern Gaging Stations Mean Monthly Discharge

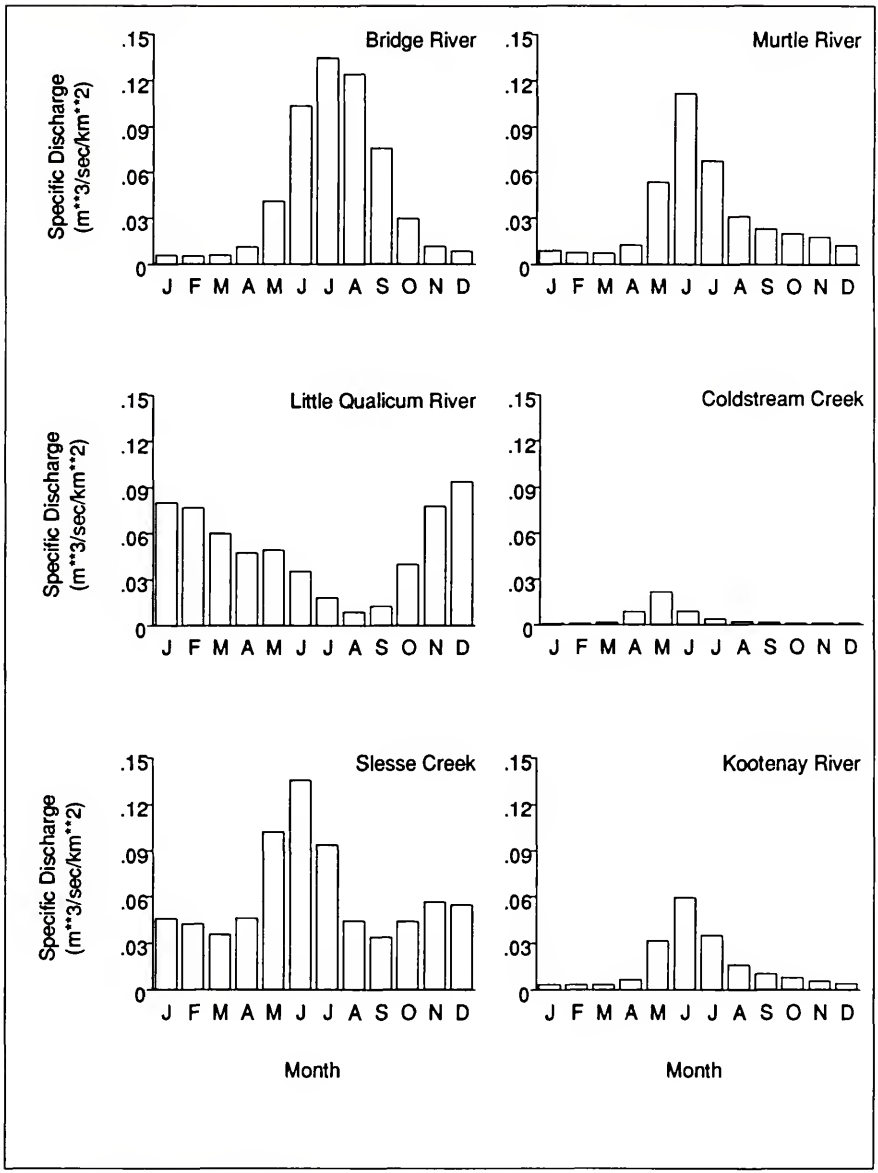

Figure 13. Southern Gaging Stations Mean Monthly Discharge

Data

The necessary daily discharge data for complete calendar years of record were obtained from the Water Survey of Canada (W.S.C.) which is responsible for the archiving and the conducting of a variety of quality control tests on the data, to ensure its accuracy. Data were obtained directly in computer compatible format on magnetic tape in cubic meters per second. (Environment Canada, 1980).

The requirements of the data and study area included 1) sampling of as wide a variety of environments and flow regimes as possible, 2) long discharge records, 3) sampling of a wide range of basin sizes, and 4) a relatively high station density, with stations sampling similarly-sized contiguous basins.

CHAPTER 4
STATISTICAL MODEL

Approach and Theory

In the calendar-year flow-duration curve both the mean discharge of each annual rank and an associated variability about that mean are retained. The empirical construction of this curve has a strong theoretical parallel to order statistics, which may be used to estimate the mean discharge at any rank once the nature of the parent distribution of all daily streamflows on the river is known.

This chapter presents the development of a statistical model for the construction and use of flow-duration curves. An outline of order statistics is provided, the theoretical model is then developed and the concept and possible forms of a parent distribution for the generation of streamflows discussed. Finally a review of the techniques employed in spatially modelling is presented.

Order Statistics

An order statistic is a ranked observation from a sample (Galambos, 1984). This is similar to the process by which daily discharges are ranked to construct a calendar-year flow-duration curve, where the sample is one year's observations. The ranking is assumed to be in descending order of magnitude throughout the research

but could equally be assumed to be ascending. There are two forms of order statistics: absolute and relative. In the first, the ranks are calculated while the size of each sample changes; for instance, the distribution of the second largest observation may be studied, regardless of sample size (so long as it is larger than one). In the second, the rank is expressed as a ratio of the sample size and the distribution of the variable at a fixed value of that ratio is considered (for example, 0.5 for the median). The ratio occupying that rank is a relative order statistic. When the sample size is fixed, as in the case of daily streamflow observations per year, excluding February 29 on leap years, these two approaches are interchangeable.

Let $X_1, X_2, \ldots, X_n$ denote a random sample from a population with continuous cumulative distribution function F_X. Within that sample, a unique ordered arrangement should exist: suppose that $X_{(1)}$ denotes the largest of that set; $X_{(2)}$ denotes the second largest, and so on; and $X_{(n)}$ denotes the smallest. Then

$$X_{(1)} > X_{(2)} > \ldots > X_{(n)}$$

denotes the original random sample after arrangement in decreasing order of magnitude, and are collectively termed the order statistics of the random sample. $X_{(r)}$, for $1 \leq r \leq n$ is called the r^{th} order statistic (Gibbons, 1985).

The annual flood, which is the maximum of the annual sample, will be denoted by $X_{(1)}$; the minimum value (the annual low-flow) will be denoted by $X_{(n)}$, and the annual median, a measure of central tendency in the sample, will be $X_{((n+1)/2)}$ or $X_{(n/2)}$ for even sample sizes.

The ensuing results from order statistics originate from large-sample approximations, and not exact solutions. It is assumed that the case of 365 daily observations is sufficiently large to constitute a large sample. Gibbons (1985) presents the general results of the large-sample approximation to the mean of the r^{th} order statistic $X_{(r)}$ of a sample n from the continuous distribution F_X. The first and second approximations to the mean, by rank are

$$\begin{aligned} \text{First Approximation: } \mu_{(r)} &= F_X^{-1}(h(r)) \\ \text{Second Approximation: } \mu_{(r)} &= F_X^{-1}(h(r)) - \\ &\quad (g(r)/2!) f_X'(F_X^{-1}(h(r))) f_X(F_X^{-1}(h(r)))^{-3} \end{aligned} \tag{1}$$

where

$$\begin{aligned} h(r) &= 1 - \frac{r}{n+1} \\ g(r) &= \frac{r(n-r+1)}{(n+1)^2(n+2)} \end{aligned} \tag{2}$$

and F_X is the cumulative distribution function, F_X^{-1} is the inverse distribution function, f_X is the density function, and f_X' is the first derivative of the density function. The second approximation provides greater accuracy than the first alone (Gibbons, 1985).

There are obvious similarities between the methodology of Gibbons (1985) and the construction of the calendar-year flow-duration curve. Estimates of $\mu_{(r)}$, the mean by rank, from order statistics theory should enable the construction of a flow-

duration curve, once the underlying frequency characteristics of the population are known.

Parent Distribution

The parent distribution is the marginal distribution of the aggregate of all the observed discharge records. The correct choice of a distribution function to represent the parent is critical, as it governs both the general form or shape of the flow-duration curve and the actual values of the curve. Generally the distribution of daily discharge is similar to those of many seasonal series and is both highly skewed and peaked. This shape is suggestive of a lognormal, gamma or a generalized extreme value distribution.

Lognormal Distributions

The choice of a lognormal frequency distribution for daily discharge is not uncommon in the literature (e.g., Ashmore & Day, 1988). There are two forms of the lognormal distribution, two parameter and three parameter. In the real data space, the distribution is positively skewed and bounded by zero on the left, which are two characteristics that make this distribution a good choice for modelling of streamflow (Haan, 1977). In the log transform domain, the data are normally distributed. The distribution is derived by the transformation $y = \log(x)$, and moment estimators (Table 1) are used to find the first two parameters giving a density function of

$$f(x)=\frac{1}{x\sigma_y\sqrt{2\pi}}e^{-\left\{\frac{(\log x-\mu_y)^2}{2\sigma_y^2}\right\}} \tag{3}$$

and a distribution function of

$$F(Y\leq y)=\int_{-\infty}^{y}\frac{1}{\sigma_y\sqrt{2\pi}}e^{-\left\{\frac{(t-\mu_y)^2}{2\sigma_y^2}\right\}}dt \tag{4}$$

The three-parameter form of the distribution is similar, except for a non-zero lower bound which can be estimated using a variety of techniques (Haan, 1977; Kite, 1977). Generally it is estimated first in real space, then the moment estimators are used on the transformed variate y = log(x-a). The three-parameter form has a density function of

$$f(x)=\frac{1}{(x-a)\sigma_y\sqrt{2\pi}}e^{-\left\{\frac{(\log(x-a)-\mu_y)^2}{2\sigma_y^2}\right\}} \tag{5}$$

and a distribution function of

$$F(Y\leq y)=\int_{-\infty}^{y}\frac{1}{\sigma_y\sqrt{2\pi}}e^{-\left\{\frac{(t-\mu_y)^2}{2\sigma_y^2}\right\}}dt \tag{6}$$

Gamma Distributions

Another positively skewed distribution is the gamma. There are two forms: a two-parameter and a three-parameter form. The two-parameter distribution is described by shape, η, and scale, λ, parameters which may be estimated from the first two moments (Table 1) (Haan, 1977; Kite, 1977). The density function of the two-parameter form is

$$f(x)=\frac{\lambda^{\eta}x^{\eta-1}e^{-\lambda x}}{\Gamma(\eta)} \qquad (7)$$
$$\text{where, } \hat{\lambda}=\bar{x}/s_x^2 \text{ and } \hat{\eta}=\bar{x}^2/s_x^2$$

The distribution function for the two-parameter form is

$$F(X\leq x)=\int_0^x \frac{\lambda^{\eta}t^{\eta-1}e^{-\lambda t}}{\Gamma(\eta)}dt \qquad (8)$$

The optional third parameter is the location parameter, γ derived from the coefficient of skewness yielding a density function (Kite, 1977)

$$f(x)=\frac{1}{\alpha\Gamma(\beta)}\left\{\frac{x-\gamma}{\alpha}\right\}^{\beta-1}e^{-\left\{\frac{x-\gamma}{\alpha}\right\}} \qquad (9)$$
$$\text{where, } \hat{\beta}=(2/\beta_1)^2,\ \hat{\alpha}=s_x/\sqrt{\hat{\beta}}\ ,\ \hat{\gamma}=\bar{x}-s_x\sqrt{\hat{\beta}}$$
$$\text{and } \beta_1=C_s\left(1+\frac{6.51}{n}+\frac{20.2}{n^2}\right)+C_s^2\left(\frac{1.48}{n}+\frac{6.77}{n^2}\right)$$

Creating a transformed variate $y=((x-\gamma)/\alpha)$, the distribution function simplifies to a simple one-parameter gamma form

$$F(Y \leq y) = \int_0^y \frac{t^{\beta-1} e^{-t}}{\Gamma(\beta)} dt \qquad (10)$$

Generalized Extreme Value Distribution

Derived by Jenkinson (1954) to include the three classical extreme value distributions, all forms are positively skewed with the primary difference in the tails of the distributions. The Type I distribution behaves as if it were unbounded in either direction, the Type II behaves as if it were bounded in the lower direction (for maxima), and Type III is bounded in the upper direction (for maxima). The value of the shape parameter, k, differentiates between the extreme value types described. For $k=0$, the distribution has the Type I form. For $k<0$, the distribution takes on the Type II form, and for $k>0$ it has the Type III form. The density functions are given as

$$\begin{aligned} f(x) &= \exp[-\{1-k(x-\xi)/\alpha\}^{1/k}](-(1/k\{1-(k(x-\xi))/\alpha\}^{(1/k-1)}(-k/\alpha) && k\neq 0 \\ &= \exp[-\exp\{-(x-\xi)/\alpha\}][-\exp(-(x-\xi)/\alpha)](-1/\alpha) && k=0 \end{aligned} \qquad (11)$$

The distribution functions are

$$\begin{aligned} F(X \leq x) &= \exp[-\{1-k(x-\xi)/\alpha\}^{1/k}] && k\neq 0 \\ &= \exp[-\exp\{-(x-\xi)/\alpha\}] && k=0 \end{aligned} \qquad (12)$$

Estimates of the parameters of this distribution can be obtained via probability weighted moments (Hosking et al., 1985)

$$\begin{aligned}\hat{k} &= 7.8590c + 2.9554c^2 \\ c &= \frac{2b_1-b_0}{3b_2-b_0} - \frac{\log 2}{\log 3} \\ \hat{\alpha} &= \frac{(2b_1-b_0)\hat{k}}{\Gamma(1+\hat{k})(1-2^{-\hat{k}})} \\ \hat{\xi} &= b_0 + \hat{\alpha}\{\Gamma(1+\hat{k})-1\}/\hat{k}\end{aligned} \tag{13}$$

$$b_r = n^{-1}\sum_{j=1}^{n} p_{j,n}^{r} x_j \tag{14}$$

where, $p_{j,n} = \frac{1}{n+1}$, and r= 0,1,2

Lognormal Mixture Distribution

An alternative to a single-component distribution for daily discharge is to use a mixture of two components as the parent. The ability of a mixture distribution to represent more complex frequency distributions is not surprising, given the necessity of estimating five parameters. The possibility that daily streamflows are drawn from mixtures of streamflow generating populations, for example, snowmelt, rainfall or storage depletion (Hirschboeck, 1985; Waylen, 1985b) is however more appealing physically.

A potentially suitable parent distribution is the two-component, two-parameter, lognormal mixture distribution (Leytham, 1984). Again, the distribution is positively skewed and bounded by zero on the left; the addition of the second component has the benefit of extending the tail of the distribution. The resulting density function for the parent of daily discharge is

$$f(y)=\rho\phi(y|\mu_1,\sigma_1^2) + (1-\rho)\phi(y|\mu_2,\sigma_2^2) \quad (15)$$

where ϕ is the two-parameter lognormal density function (Equation 3) and ρ is the mixing proportion of the first component. The distribution function is

$$F(Y\leq y)=\rho\Phi(y|\mu_1,\sigma_1^2) + (1-\rho)\Phi(y|\mu_2,\sigma_2^2) \quad (16)$$

where Φ is the two-parameter lognormal distribution function (Equation 4). A primary advantage of this particular distribution is that estimation of the parameters is not dependent on an a priori classification scheme. Leytham (1984) provides a complete maximum likelihood estimation procedure using the expectation-maximization algorithm of Dempster et al. (1977).

$$\begin{aligned}
w_{i,j}^{(v)} &= [p_j^{(v)} f_j(y_i|\phi^{(v)})] \left[\sum_{k=1}^{2} p_k^{(v)} f_k(y_i|\phi^{(v)})\right]^{-1} \\
&\quad i=1,\ldots,n \quad j=1,2 \\
p_j^{(v+1)} &= n^{-1}\sum_{i=1}^{n} w_{i,j}^{(v)} \\
&\quad j=1,2 \\
\mu_j^{(v+1)} &= \left[\sum_{i=1}^{n} w_{i,j}^{(v)} y_i\right]\left[\sum_{i=1}^{n} w_{i,j}^{(v)}\right]^{-1} \\
&\quad j=1,2 \\
\sigma_j^{(v+1)} &= \left\{\left[\sum_{i=1}^{n} w_{i,j}^{(v)}(y_i-\mu_j^{(v)})^2\right]\left[\sum_{i=1}^{n} w_{i,j}^{(v)}\right]^{-1}\right\}^{1/2} \\
&\quad j=1,2
\end{aligned} \quad (17)$$

The transformed variate is $y=\ln(x)$, μ_1 and μ_2 are the means of the respective processes, and σ^2_1 and σ^2_2 are the variances of the individual components. Given the structure of the distribution, the first component always has the smaller mean.

Table 1. Common Moment Estimators

$$\text{Mean, } \mu_x = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$\text{Variance, } \sigma_x^2 = \frac{1}{n-1}\sum_{i=1}^{n} (x_i-\mu_x)^2$$

$$\text{Skewness, } \gamma_x = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n} (x_i-\mu_x)^3$$

$$\text{Coefficient of skewness, } C_s = \frac{\gamma_x}{\sigma_x^3}$$

$$\text{Coefficient of variation, } C_v = \frac{\sigma_x}{\mu_x}$$

Spatial Analysis

Regionalization

The current use of the term "regionalization", in hydrology, refers to the "substitution of space for time" in improving the estimation of the probabilities of extreme floods (National Research Council, 1988). Regionalization is a form of the classification of data which have been collected for spatial units into measurably homogeneous regions. Regionalization based on maps of homogeneity statistics such as the coefficient of variation provides a subjective interpretation of the spatial patterns, and the delimitation of regions of a continuous variable becomes an art. Cluster analysis provides an objective approach to regionalization based on

parameters of the parent distribution. Groups of rivers whose parameters are similar compose hydrologic regions.

Cluster analysis is a generic name given to a set of various procedures used to create a classification scheme (Aldenderfer & Blasfield, 1984). As such there are a few potential problems: 1) most methods are empirically based, and not supported by theory, 2) different methods generate different resultant clusters from the same data set, and 3) although used to discover structure in data, their optimal use requires recognition of physically meaningful groupings, as opposed to those which are merely an artifact of the numerical procedures (Aldenderfer & Blasfield, 1984).

Spatial Estimation

Spatial estimation of parameter values at locations with absent or insufficient observations is a problem in applied geography. The construction of maps of parameter values is complicated by the need to interpolate between observations in order to facilitate prediction and may be accomplished through a variety of algorithms. Figure 14 shows a sample grid from a mapping program such as SURFER (Golden Software Inc., 1989). The program starts with a series of random observations as marked on the map, the grid is superimposed and the program estimates the values of the variable at each grid intersection. Usually, the grid cell is used as the search radius around the intersection, and observations within that radius are averaged to estimate the value for a point; when no observations are available nearby, the estimation of the value at a grid intersection is accomplished

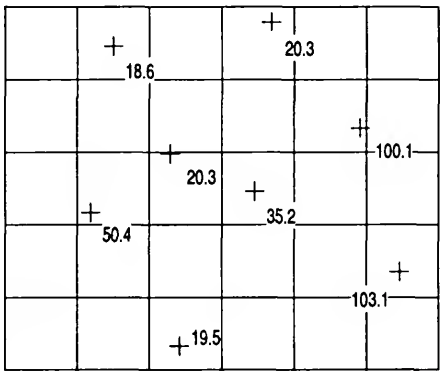

Figure 14. Sample Grid

with the gridding algorithm. Golden Software's SURFER program provides three techniques for mapping:

1) The inverse distance algorithm simply weights each data point's influence on another by the inverse of the distance raised to a specified power and sums over the requisite number of nearest neighbors; this tends to be very fast computationally, but lacks analytical sophistication.

2) The minimum curvature method fixes the value of a grid point with an observation, if present, and then repeatedly smooths the surface with a mathematical equation until a specified minimum error is achieved. This technique is very attractive because it ensures that observations are honestly reflected in the resulting map.

3) Kriging uses regional variable theory to provide a minimum variance unbiased linear estimator of the mean characteristic of a grid cell for a given geometry (Golden Software Inc., 1989; Journel & Huijbregts, 1978). In practice however, the results of kriging depend on the skill of the person using the software and their familiarity with the technique.

Inverse distance procedures tend to be very sensitive to the search procedure employed to include data points in the estimation. Expanding the search to quadrants or octants introduces a much higher degree of smoothing to the result. A nearest neighbor search on the other hand may generate unrealistic trends or slopes in areas of limited data, and translate poorly to the estimation of grid points (Golden Software Inc., 1989).

While both the minimum curvature and kriging techniques are said to be more accurate because they project trends in the data (Golden Software Inc., 1989), the opposite may in fact be the case. Spurious highs or lows in areas of limited data may result if trends are projected from areas of tightly clustered control points (Sampson, 1978). Kriging also requires prior determination of the semivariogram, a function relating covariance between points to the distance between the points. The effectiveness of kriging depends on the proper selection of parameters, including the semivariogram (Sampson, 1978). SURFER has a linear semivariogram built-in; this particular choice is extremely poor as it implies that variance increases without bounds with increasing distance (Sampson, 1978). Additionally, the choice of underlying drift is imperative as all estimates depend on this trend; again this requires prior investigation.

Concluding Remarks

Application of the order statistic-based model begins with identification of the parent distribution and the subsequent estimation of it's parameters. Each of the distributions outlined in this chapter have the correct shape for the distribution of daily streamflow. The final choice for the parent distribution can be made on the basis of goodness-of-fit tests and the physical reasonableness of the distribution.

Cluster analysis allows objective identification of homogeneous regions in the parameters of the parent distribution. Results of this regionalization may be interpreted using the known underlying physical geography of the study area, and

compared to other regionalization efforts in the same area, based on such variables as the mean annual flood.

Using the minimum curvature technique, parameters of the parent distribution of streamflow for each basin may be mapped. Specification of the degree of error allowed in estimation of the surface, and the requirement that observations be honestly presented in the final gridded surface make this the best choice for the study area with it's high degree of underlying variability. The resultant maps provide for the estimation of the parameters of the parent distribution at any location.

CHAPTER 5
RESULTS

The statistical model of flow-duration curves is applied to all rivers in the study area, a subset of which are chosen and discussed in detail. Results are presented as a series of maps of each of the parameters of the chosen parent distribution. The physical basis of the observed variation of the parameters are discussed, and the results of cluster analysis on the parameters of the parent distribution are presented. A set of predictions are made from the maps, and compared to limited historic records at these sites.

Four rivers possessing widely differing flow regimes are selected to illustrate the performance of the model (Figure 15). The Sumas River's flow regime is dominated by flows generated from winter rains and to a lesser extent smaller summer convective storms. The basin, situated entirely on the Fraser Lowlands, is relatively small with an area of 149 km^2 and possesses little lake storage. The regime of Slesse Creek possesses two periods of high flow, one during spring snowmelt and the other resulting from large winter rains. It is a small (162 km^2) mountainous basin on the windward side of the Cascade Mountains, with little natural storage in the form of lakes and lowlands. The Kitimat River drains a mountainous basin of 1,990 km^2 in the Coast Mountains. High flows are generated by spring snowmelt, summer

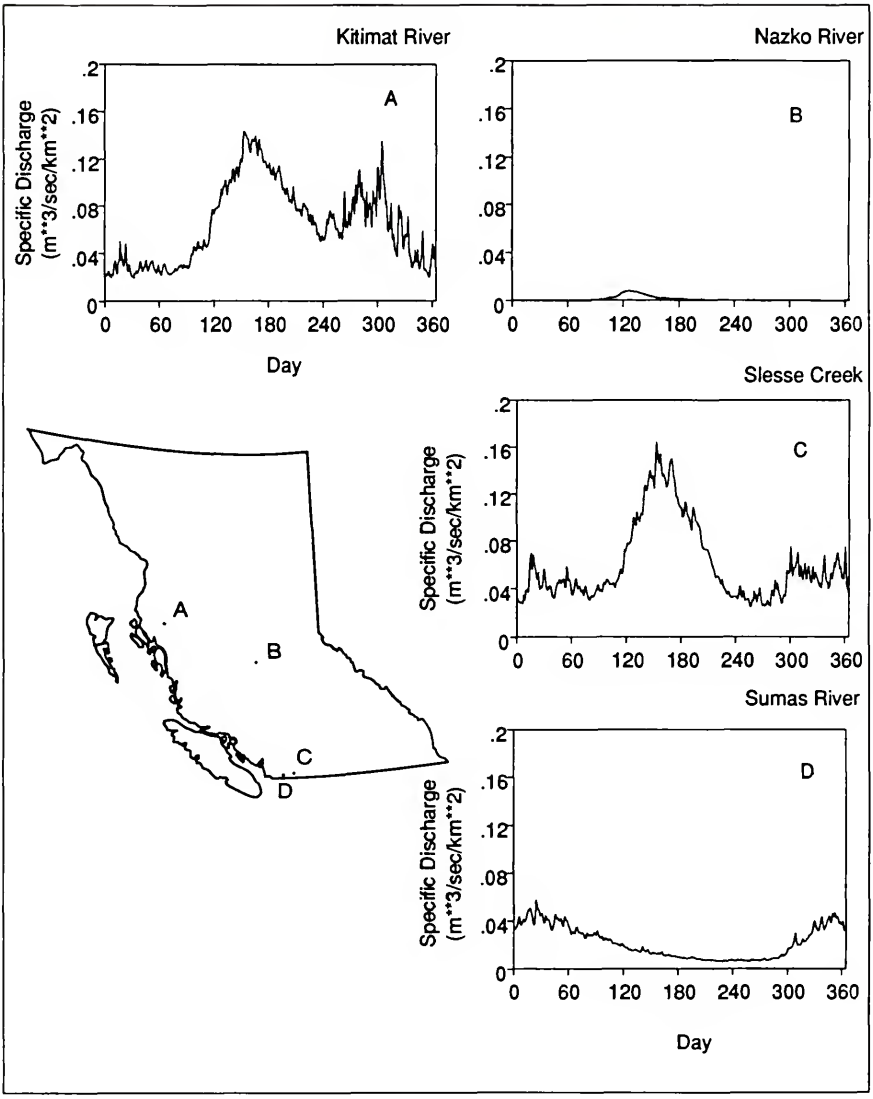

Figure 15. Study Rivers, Gage Locations and Average Annual Hydrographs

glacier melt and fall rains; there is little lake storage within the basin. The last river, the Nazko, located on the Nechako Plateau, characterizes the dry interior regimes. There is a single high flow period of spring snowmelt, and the rest of the year is generally dry with a large winter draw-down following the fall freeze-up. The Nazko is the largest of the four basins studied, with an area of 3,240 km^2.

Statistical Model

The distributions of observed flows (Figure 16) in the four sample rivers are highly skewed and peaked, with particularly accentuated upper tails. The discharge classes used in Figure 16 are derived by dividing the observed range of the data into twenty equal classes. The probability distributions listed in Chapter 4 are estimated and fit to the observed data at all one hundred and eighty two stations. The results of the application of the standard Kolmogorov-Smirnov and Chi-squared goodness-of-fit tests to the cumulative and density forms, respectively, of the four sample records are summarized in Tables 2 and 3. Not surprisingly, the goodness-of-fit statistics are generally better using a 2-component lognormal mixture distribution as the parent. The distribution is capable of representing both the small inflation of the frequency distribution at the high-discharge end of the curve and the relative peakedness of the lower-discharge end in all four diverse hydrologic environments (Figure 16).

Assuming the mixed lognormal distribution to be an adequate parent, expected flow-duration curves are generated and visually compared to the observed curves in Figure 17. As the flow-duration curve represents the mean discharge at a

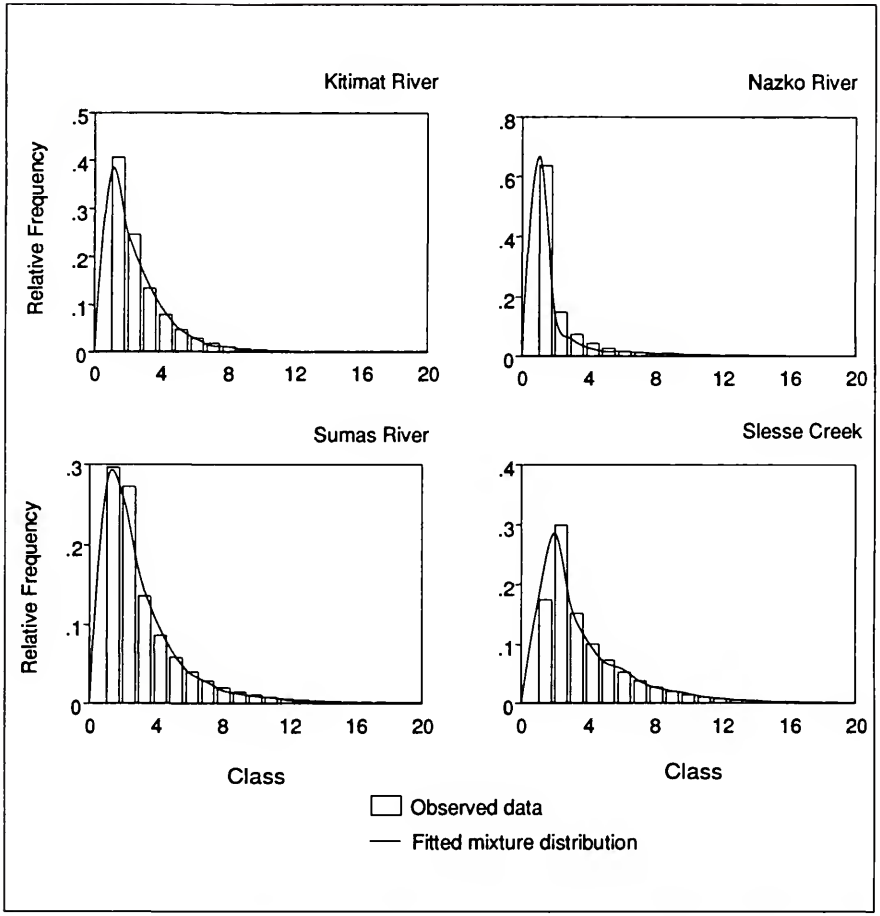

Figure 16. Study Rivers, Parent Frequency Distributions

Table 2. Parameter Estimates for Parent Distributions

Distribution		Sumas R.	Nazko R.	Kitimat R.	Slesse C
Two-Parameter Lognormal	μ_y	-4.22369	-7.23511	-3.09988	-3.10808
	σ_y	0.90381	1.08970	0.88433	0.76768
	K-S	0.01578	0.11007	0.03964	0.04433
	χ^2	16.2	521.9	136.8	159.3
Three-Parameter Lognormal	μ_y	-4.23046	-7.25352	-3.13984	-3.13729
	σ_y	0.9115	1.10414	0.92159	0.78794
	a	0.00006	0.000008	0.00116	0.00097
	K-S	0.01627	0.1076	0.04394	0.04073
	χ^2	19.9	492.3	176.0	144.6
Two-Parameter Gamma	λ	42.7286	214.014	17.6439	21.5224
	η	0.92871	0.32126	1.14080	1.30195
	K-S	0.12034	0.33926	0.06431	0.10713
	χ^2	168.7	1241.7	89.7	137.7
Three-Parameter Gamma	β	0.35026	*	0.29685	0.7555
	α	0.03811		0.11111	0.061
	γ	0.00839		0.03168	0.01441
	K-S	0.27909		0.33577	0.05114
	χ^2	376.2		597.6	8.8
Generalized Extreme Value	k	-0.32642	-0.60844	-0.23191	-0.29767
	α	0.00975	0.00049	0.03153	0.02577
	ξ	0.01153	0.00049	0.03718	0.35011
	K-S	0.04155	0.09902	0.04011	0.0518
	χ^2	65.9	333.8	32.7	275.3
Sample size		11680	7300	7665	8760
Years of record		32	20	21	24
Number of Classes		20	20	20	20

* Unable to fit a three-parameter gamma distribution for this river.

Table 3. Mixture Distribution Parameters for Parent Distribution

Station	Sample Size	Parameter	Observed Values*	K-S Statistic	χ^2 Statistic
Sumas R.	11680	μ_{y1}	-4.94953	0.04264	187.383
		σ_{y1}	0.490131		
		ρ	0.368228		
		μ_{y2}	-3.80063		
		σ_{y2}	0.816581		
Nazko R.	7300	μ_{y1}	-7.98199	0.04308	165.288
		σ_{y1}	0.481234		
		ρ	0.431945		
		μ_{y2}	-6.66719		
		σ_{y2}	1.08041		
Kitimat R.	7665	μ_{y1}	-3.86103	0.03933	139.778
		σ_{y1}	0.553602		
		ρ	0.348457		
		μ_{y2}	-2.69281		
		σ_{y2}	0.748783		
Slesse C.	8760	μ_{y1}	-3.72620	0.0168	34.7307
		σ_{y1}	0.393342		
		ρ	0.394296		
		μ_{y2}	-2.70569		
		σ_{y2}	0.679338		

* in log space

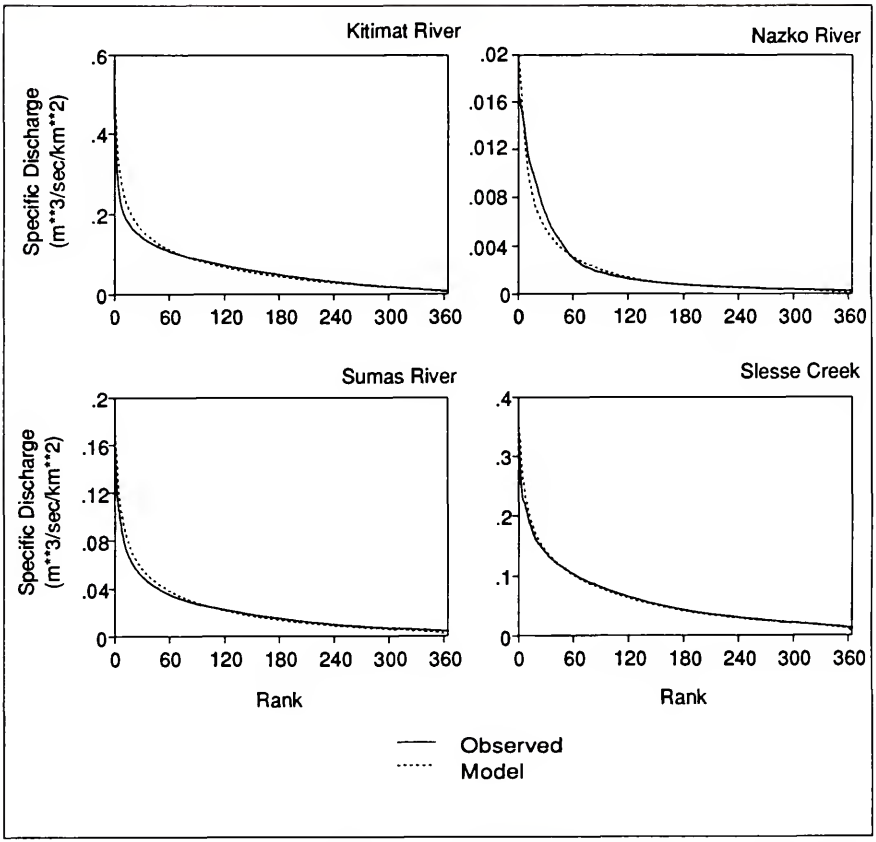

Figure 17. Study Rivers, Observed and Model Flow-Duration Curves

particular rank it is reasonable to perform one-sample t-tests of significant difference between observed and predicted data. The necessary assumption of normality is theoretically supportable at intermediate and central ranks, but not at the extreme ranks (Leadbetter et al., 1983). It is only at the latter ranks that the observed data and fitted models can be considered significantly different at the 0.05 level.

Regionalization and Prediction

Results

The mixed lognormal distribution parent is fit to the 182 stations in Figure 18 and the estimated parameters are listed in Appendix B. Parameters are estimated in log space and converted to real space by the following transformations (Kite, 1977)

$$\begin{aligned} \bar{x} &= \exp[\bar{y}+s_y^2/2] \\ s_x^2 &= \bar{x}^2[\exp(s_y^2)-1] \end{aligned} \tag{18}$$

for each of the two sub-populations in the mixture. Maps of each of the five parameters in real space (Figure 19-23), except for the mixing proportion (Figure 21) reflect the importance of annual precipitation (Figure 5). In all cases, the Coast and Columbia mountain areas are prominent north west/south east features, separated by the interior plateau.

Larger values for both the mean (Figure 19) and standard deviation (Figure 20) of the smaller sub-population are observed along the coast. The mean is greater

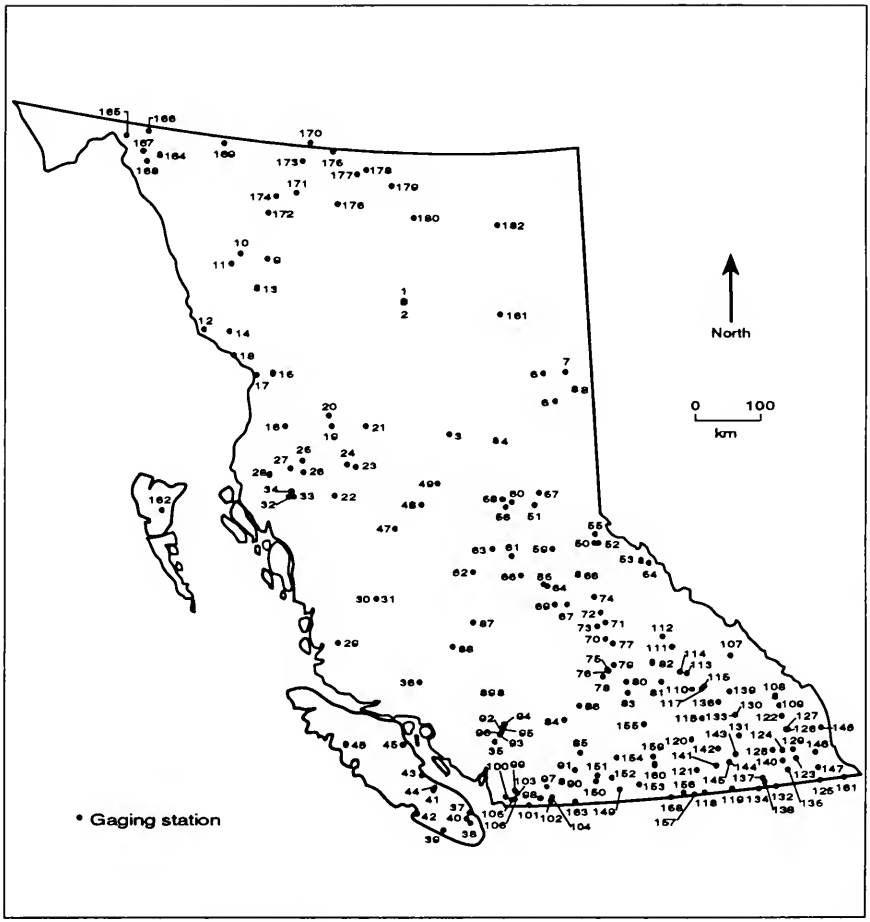

Figure 18. Hydrometric Network

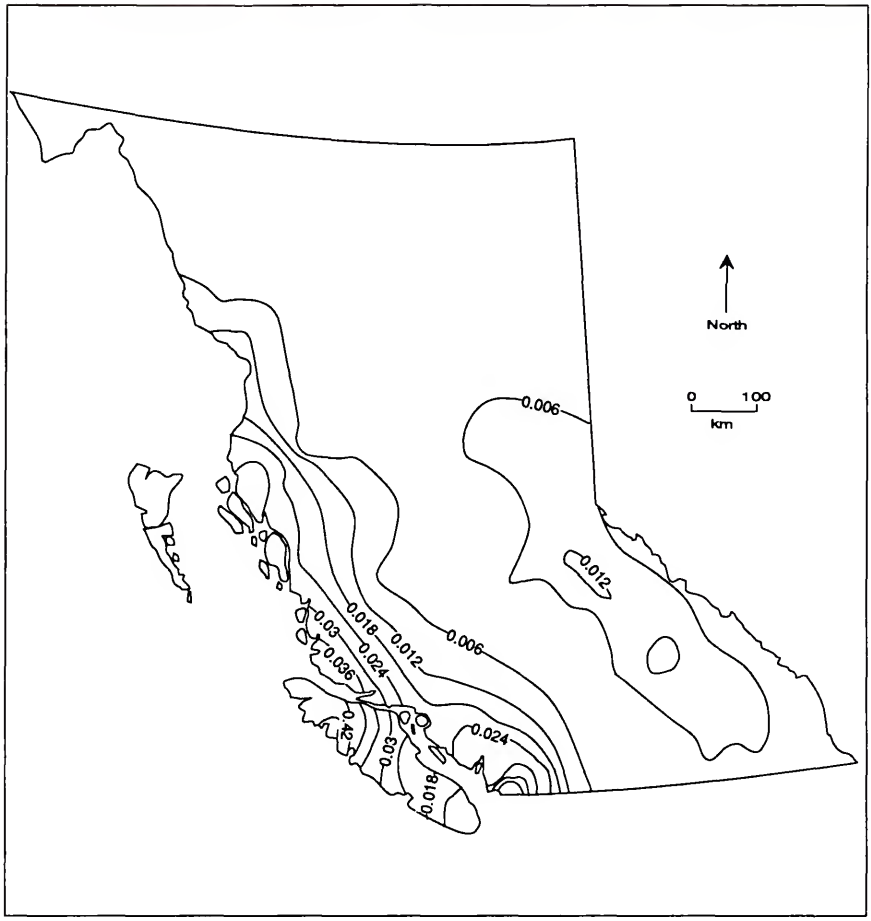

Figure 19. Mean of the First Component (Contour Interval = 0.006 $m^3s^{-1}km^2$)

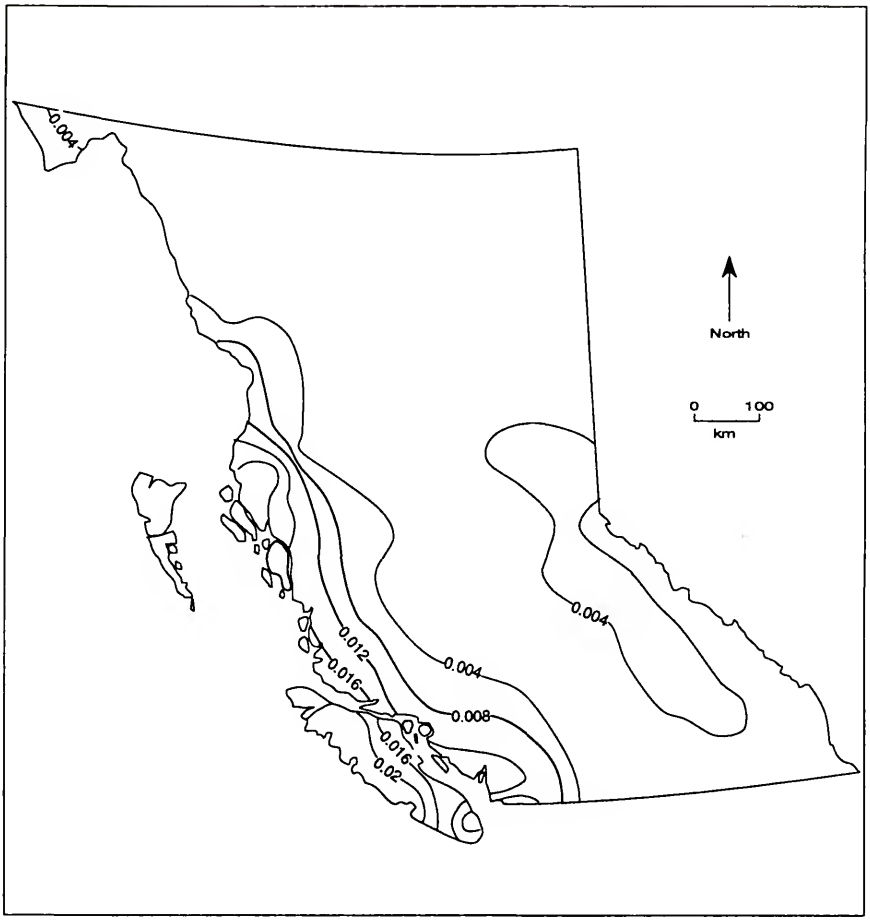

Figure 20. Standard Deviation of the First Component (C.I. = 0.004 $m^3s^{-1}km^2$)

than 0.036 $m^3s^{-1}km^2$ and the standard deviation above 0.02 $m^3s^{-1}km^2$. Physically these correspond to summer rains in littoral locations. Inland, the smaller process is draw-down of storage within a basin during winter. The leeward side of the Coast Mountains and Columbia Mountains are higher in both mean and standard deviation than the dry interior (mean and standard deviation less than 0.006 and 0.004 $m^3s^{-1}km^2$ respectively), Alberta Plateau and the Rocky Mountain trench.

Two sub-parallel ridges of high values corresponding to mountain ranges appear in values of the mean (above 0.1 $m^3s^{-1}km^2$)(Figure 22) and standard deviation (0.1 $m^3s^{-1}km^2$)(Figure 23) of the larger process. These are generally an order of magnitude larger than the smaller component. Along the coast, the larger process is generated by winter rains and rain on snow. In the interior, the larger process results from spring snowmelt and possibly summer glacier melt. The dry interior and Alberta Plateau and Rocky Mountain trench all stand out clearly. The steep gradient of processes from the windward side of the Coast Mountains to the dry interior, and to a lesser extent for the Columbia Mountains is a dominant feature of all these maps.

Figure 21 depicts the mixing proportion of the smaller process and is more difficult to interpret, partially because the physical processes correspond to the smaller sub-population varies across the study area. In general, the proportion of time that the smaller process is active increases to the south and east (from 0.26 to 0.42 and 0.39 to 0.45, respectively). The larger this parameter, the more restricted is the period of high flows.

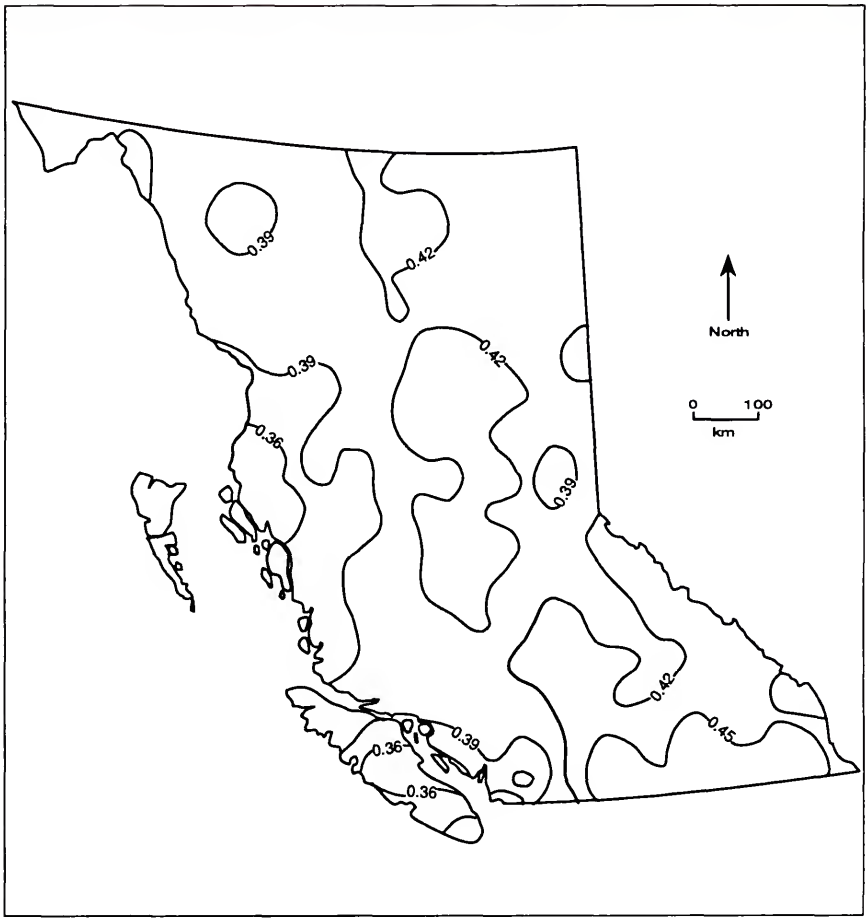

Figure 21. Mixing Proportion (C.I. = 0.03)

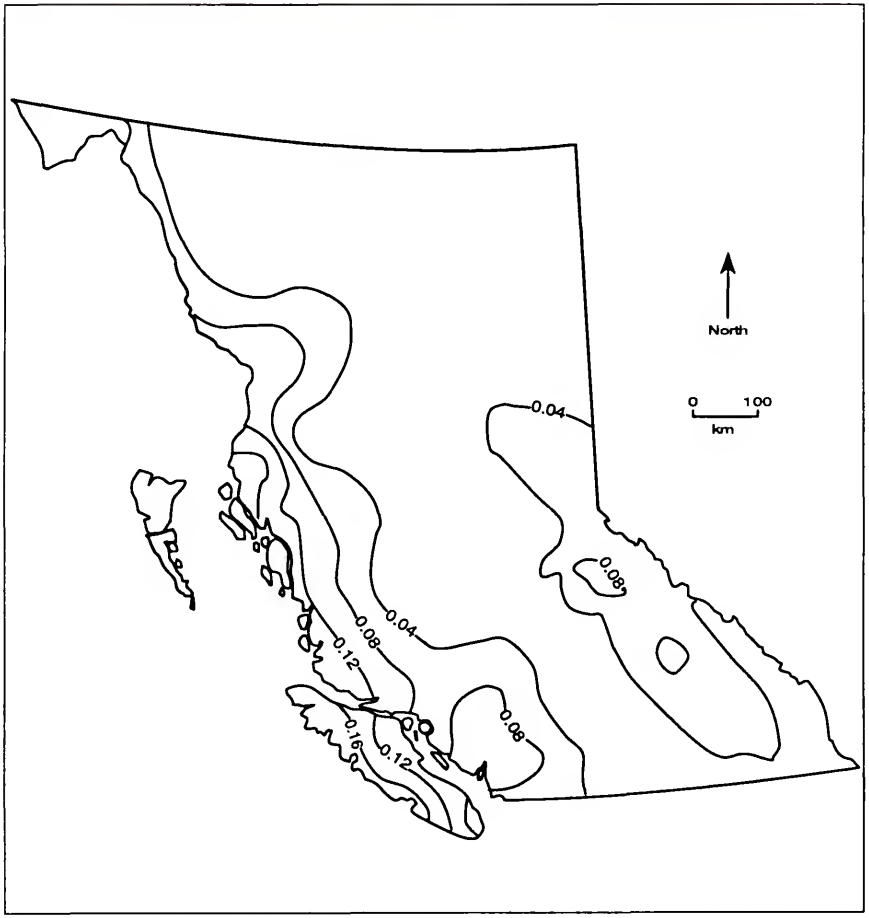

Figure 22. Mean of the Second Component (C.I. = 0.04 $m^3s^{-1}km^2$)

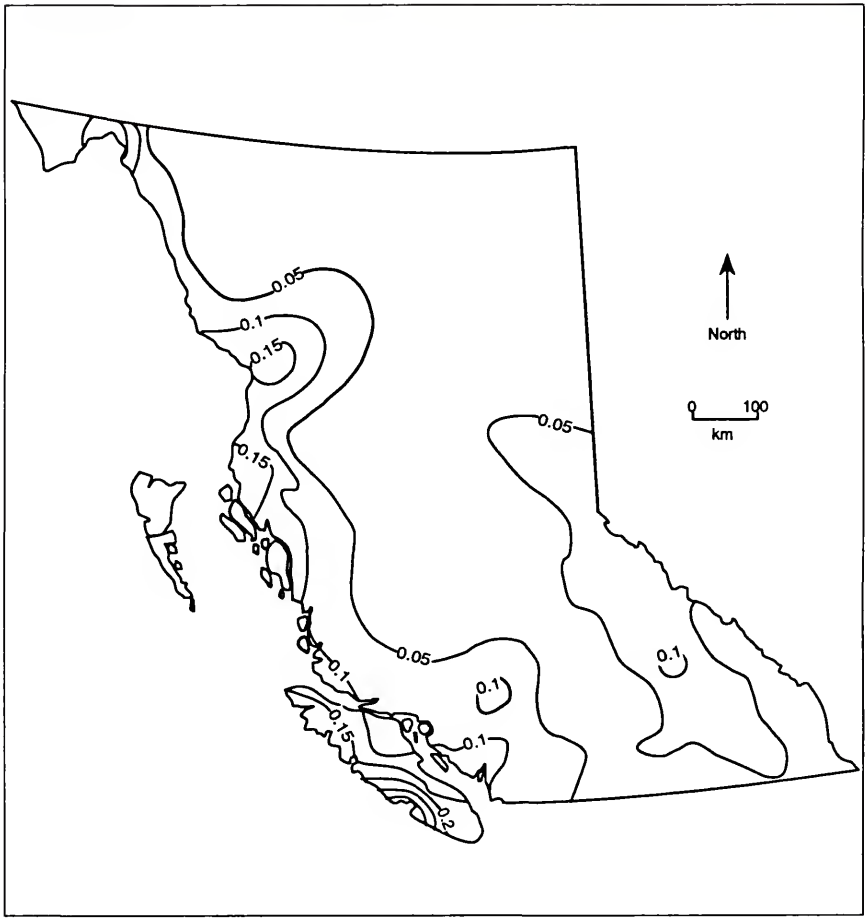

Figure 23. Standard Deviation of the Second Component (C.I. = 0.05 $m^3s^{-1}km^2$)

Spatial Analysis

Figures 19 through 23 suggest certain regions of similar hydrologic behavior as indicated by the parameters of the mixture distributions. Objective groupings of basins possessing similar sets of parameters are identified using the cluster analysis procedures within SAS (SAS Institute Inc., 1988). Two-stage density linkage cluster analysis was used to identify seven relatively homogeneous clusters. This particular technique of analysis is chosen because the algorithm ensures that all points are assigned to modal clusters before modal clusters themselves are allowed to join, and few constraints are placed on the data. The derived cluster membership is depicted in Figure 24. The average flow-duration curves for each cluster are constructed in Figure 25, and the parameters in each cluster are summarized in Table 4.

Clusters II and III are representative of a general type of flow-duration curve. The means and standard deviations of both components are small (less than 0.005 and 0.002, respectively), and the mixing proportion of the smaller process is relatively large (0.41 for II, and 0.46 for III). These flow-duration curves are typical of snowmelt dominated flow regimes. Both clusters have the longest period of monthly minimum temperatures below freezing, averaging 6.3 months. Cluster III has the lowest total annual precipitation of all clusters with less than 500mm.

Cluster III is located geographically in the south central part of the province in the driest area. This is reflected by the consistently small values of discharge in the flow-duration curve. The regime represented is spring snowmelt and storage

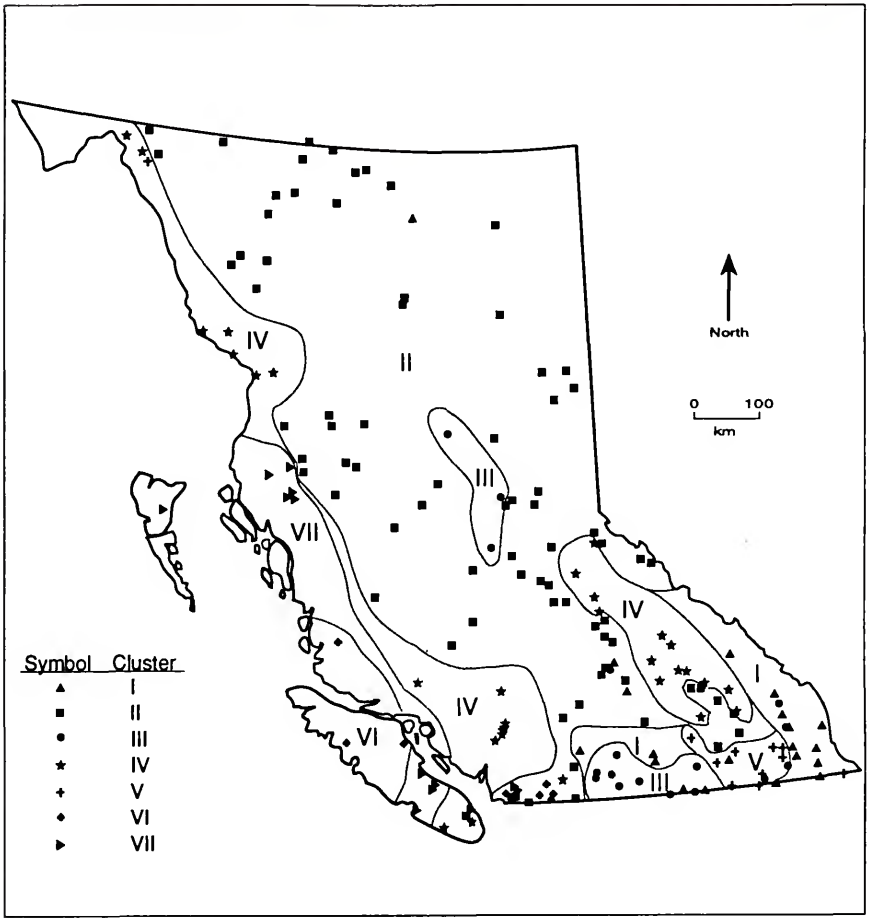

Figure 24. Cluster Membership and Regions

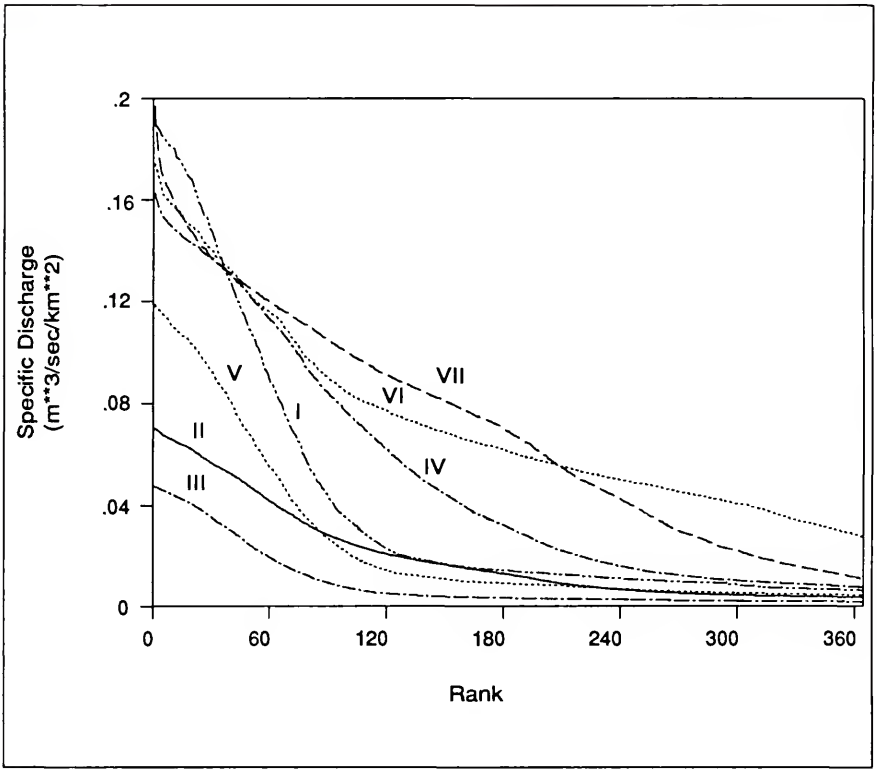

Figure 25. Average Flow-Duration Curves by Cluster Membership

Table 4. Cluster Summary of Parameters of the Mixture Distribution Parent

Cluster	Number of Gages	Parameter	Mean	Standard Deviation
I	22	μ_{x1}	0.0041102	0.0015714
		σ_{x1}	0.0016596	0.0004972
		ρ	0.4435570	0.0057512
		μ_{x2}	0.0258839	0.0058879
		σ_{x2}	0.0365977	0.0153759
II	79	μ_{x1}	0.0054826	0.0031371
		σ_{x1}	0.0026180	0.0015016
		ρ	0.4116332	0.0181721
		μ_{x2}	0.0319591	0.0165782
		σ_{x2}	0.0363247	0.0222834
III	17	μ_{x1}	0.0023026	0.0012310
		σ_{x1}	0.0009005	0.0003964
		ρ	0.4619863	0.0072412
		μ_{x2}	0.0155354	0.0063279
		σ_{x2}	0.0245944	0.0110716
IV	32	μ_{x1}	0.0119054	0.0041608
		σ_{x1}	0.0065709	0.0026482
		ρ	0.4065077	0.0202680
		μ_{x2}	0.0849624	0.0188469
		σ_{x2}	0.1050167	0.0364385
V	11	μ_{x1}	0.0056481	0.0015614
		σ_{x1}	0.0024117	0.0005285
		ρ	0.4565075	0.0080696
		μ_{x2}	0.0417693	0.0061117
		σ_{x2}	0.0658142	0.0159323
VI	8	μ_{x1}	0.0315927	0.0105845
		σ_{x1}	0.0127372	0.0051053
		ρ	0.3905571	0.0161944
		μ_{x2}	0.0976899	0.0417523
		σ_{x2}	0.0731671	0.0457388
VII	13	μ_{x1}	0.0202548	0.0059838
		σ_{x1}	0.0157667	0.0037205
		ρ	0.3461062	0.0136073
		μ_{x2}	0.1094643	0.0365555
		σ_{x2}	0.1375107	0.0691916

depletion through the remainder of the year. Cluster II is also a snowmelt-type curve with a slightly larger spring snowmelt and extensive period of storage draw-down. The second cluster is spread out through all of the dry interior and fans out into most of the northern extent of the province.

Clusters I and V have very large second component means (0.03 and 0.04), but the mixing proportion of the smaller process (0.44-0.45) indicates that the flow regime represented is dominated by storage depletion. Annual precipitation is higher in these clusters, averaging 600mm and the winter appears to be shorter, averaging 5.8 months with monthly minimum temperatures below freezing.

Cluster V is a transition cluster: the larger process is greater than the two previous mentioned clusters, but the smaller process is active longer in the year. Cluster I is very similar in the low-discharge end of the curve to the predominately winter draw-down processes of the previous clusters. This cluster represents a transitional type, from clusters with a very small winter process which, acts for a long period of the year, to clusters with a very large process which acts for a short period of the year. As well, the flattening at the top of the curve indicates the presence of storage acting on the highest flows.

Clusters IV, VI and VII are dominated by very large second processes (means of 0.09, 0.1 and 0.11) and larger first components than the two previous groups (means 0.01, 0.03 and 0.02). Differences between the flow-duration curves of clusters within this group are represented more by the importance and size of the smaller

process. These clusters have an average total precipitation of around 1700mm, almost entirely as rainfall, and the shortest period of freezing temperatures.

Cluster IV is found along the top of the Columbia Mountains and the leeward side of the Coast Mountains. These rivers have a very consistent decrease in the curve through all ranks, indicative of a consistent variability through the year, perhaps reflecting the presence of glaciers and large semi-permanent snow packs. Cluster VII has the largest process in the high discharge end of the curve. Rivers that make up this cluster appear to be found along the tops of the Coast Mountains. Again the relatively slow decline of the curve to the low-discharge end shows the presence of semi-permanent storage such as snow packs. Cluster VI has the highest low-flows observed in all clusters, as well as the most persistent inflation of the curve. These rivers are located low along the windward side of the Coast Mountains and, just above and interior to, the Fraser River delta. The differences between these three mountainous clusters is best reflected by average climatological conditions: Cluster IV has an average total precipitation of 1200mm and approximately 5.3 months of monthly minimum temperatures below freezing and the other two clusters having precipitation totals averaging 2000mm and approximately 2.3 months with minimum temperatures below 0°C.

Summary

The model for flow-duration curves yields parameters which are physically interpretable and appear reasonable, with regard to the hydrologic processes active within the study area. The resulting distribution for the parent of the generating processes of streamflow is capable of representing the flow-duration curves for a wide variety of hydrologic environments. Analysis of the spatial distribution of the parameters of the parent distribution shows spatially contiguous regions and therefore it is reasonable to assume that predictions are possible.

Predictions

Records on four rivers, located in differing physical environments and previously unused in the analysis, are selected as test stations for the spatial estimation of the parameters of the parent distribution. The locations and annual hydrographs, based on the available record are shown in Figure 26. Summary of the location and record lengths of these four stations may be found in Appendix C. The regime of the Atnarko River, on the leeward side of the Coast Mountains, is dominated by spring snowmelt and rare fall precipitation. The West Road River lies entirely within the dry interior, it's regime dominated by small spring snowmelt peak, and long recession through summer and winter. The Mamquam River drains the windward side of the south west region of the Coast Mountains and has high runoff from spring snowmelt, summer glacier melt and both fall and winter rains. Grave Creek is a snowmelt dominated flow regime in the Rocky Mountains.

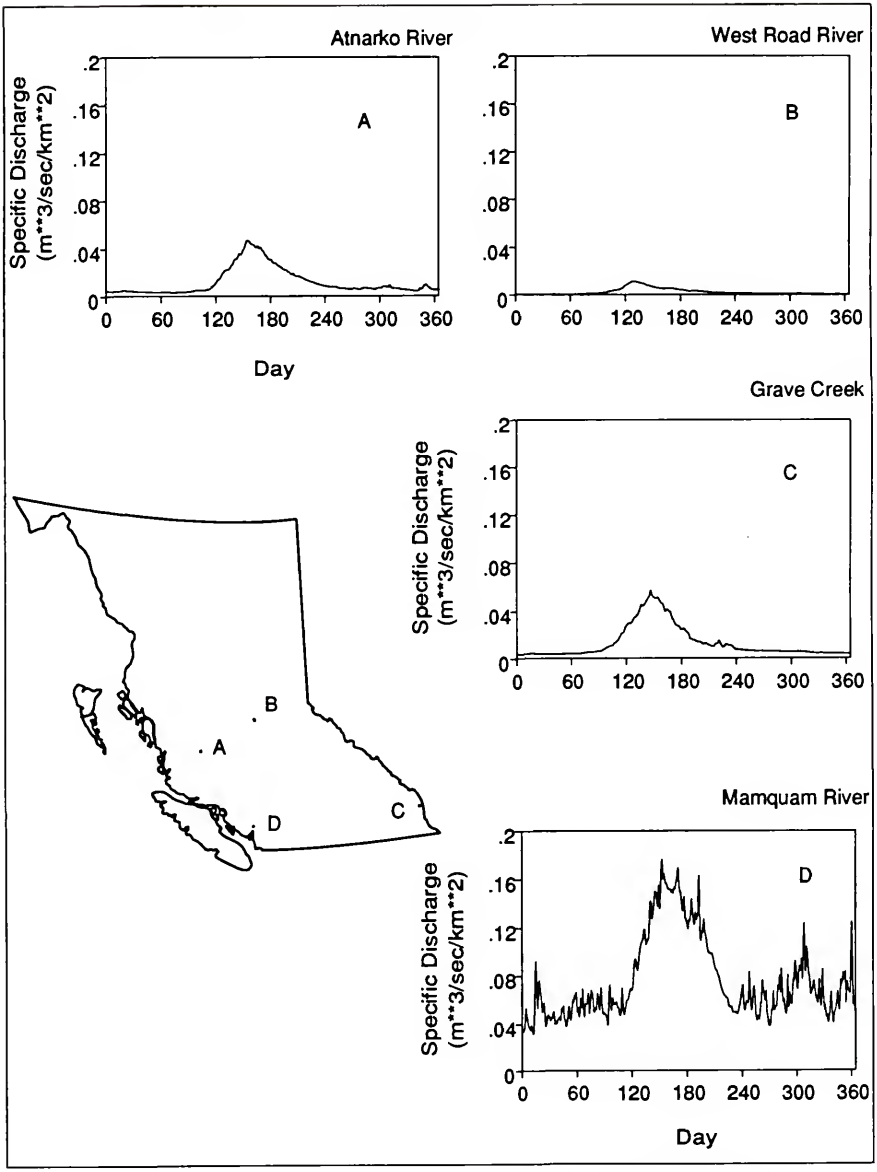

Figure 26. Test Rivers, Gage Locations and Average Annual Hydrographs

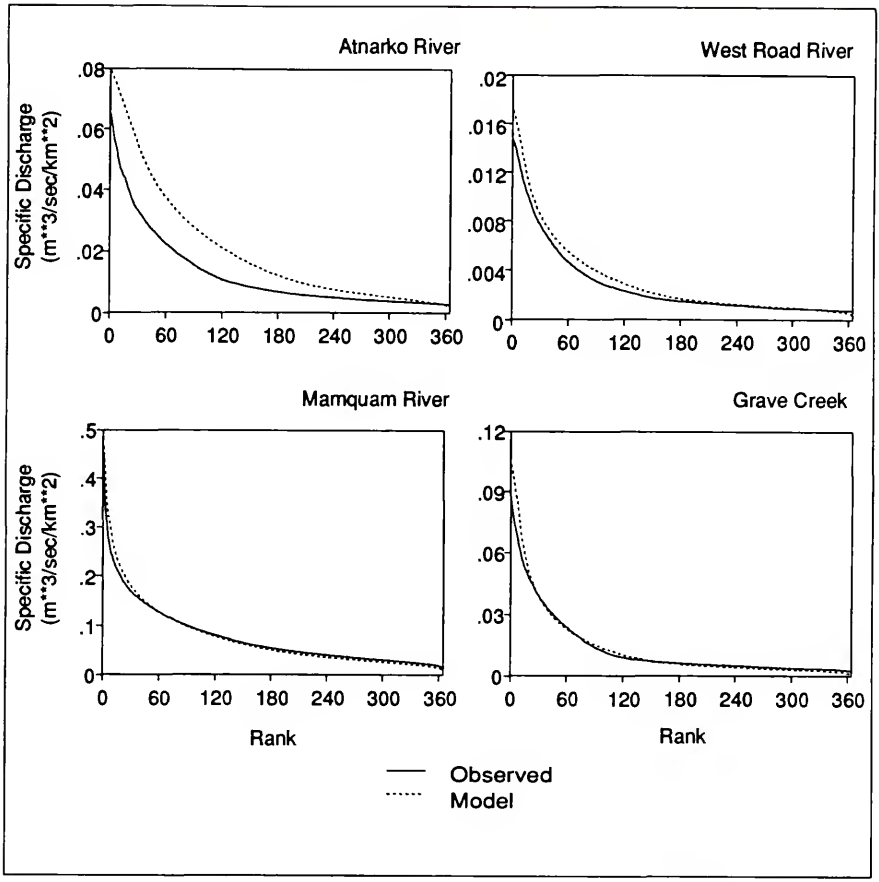

Figure 27. Test Rivers, Observed and Predicted Model Flow-Duration Curves

The five parameters of the parent mixture distribution are estimated by interpolation from more detailed versions of Figures 19 - 23. These numbers are converted to log space using the following equations (Kite, 1977)

$$\begin{aligned} \bar{y} &= 1/2 \ln[\bar{x}^2/(C_v^2+1)] \\ s_y^2 &= \ln(C_v^2+1) \\ \text{where, } C_v &= s_x/\bar{x} \end{aligned} \qquad (19)$$

Flow-duration curves in Figure 27 are generated from the resultant parent distribution parameters (Table 5). There is an obvious mis-specification for the Atnarko River. Parameter estimates of both the larger and smaller processes yield a curve with the correct shape, but which consistently over estimates the mean flow at each rank. Performance on the remaining three rivers is better: in general, estimates for the smaller process are very close to observed values, while the larger process is consistently over-estimated. Grave Creek and the Mamquam River have reasonable estimates for the mean at high ranks, but the larger component has parameter estimates which translate to the over-estimation of the mean for low ranks. However, the shape of the curves match that of the observed data. The West Road River shows over-estimation of the parameters of both components and consistent overestimation of the mean by rank, for all ranks. Assuming a normal distribution for all ranks, t-tests are used with mixed results. There are regions of significant differences between observed and predicted means at the 0.05 significance level for all four rivers. The highest ranks (low discharges) for all four are

Table 5. Map Predictions of Parameters of the Mixture Distribution Parent

Station	Parameter	Interpolated Values	Estimated Values	Percent Error
West Road River	μ_{x1}	0.00118	0.001047	12.7
	σ_{x1}	0.00041	0.000312	31.4
	ρ	0.453	0.458530	-1.2
	μ_{x2}	0.005	0.004064	23.0
	σ_{x2}	0.005	0.003953	26.5
Atnarko River	μ_{x1}	0.00648	0.004462	45.2
	σ_{x1}	0.0029	0.001714	69.2
	ρ	0.4133	0.432598	-4.5
	μ_{x2}	0.0318	0.018045	76.2
	σ_{x2}	0.0288	0.017054	68.9
Mamquam River	μ_{x1}	0.0321	0.035405	-9.3
	σ_{x1}	0.01218	0.012557	-3.0
	ρ	0.3978	0.404934	-1.8
	μ_{x2}	0.1063	0.105372	0.9
	σ_{x2}	0.0831	0.076681	8.4
Grave Creek	μ_{x1}	0.00409	0.004872	-16.1
	σ_{x1}	0.00154	0.001505	2.3
	ρ	0.4552	0.463745	-1.8
	μ_{x2}	0.0225	0.019906	13.0
	σ_{x2}	0.0303	0.021319	42.1

significantly different, despite their apparent visual fit. The remainder of all curves show fragmented areas where the t-tests indicate significant differences. The Mamquam River has 200 ranks with significant differences, predominantly at the lowest discharges. The West Road River has the overall best fit with 70 ranks at intermediate and extreme low discharge ranks showing significant differences. Grave Creek has the bottom 150 ranks showing as significantly different, and the Atnarko has all ranks except for the lowest discharges, as being significantly different.

Conclusions

The statistical model of flow-duration curves performs very well for rivers where the parameters of the parent distribution may be estimated directly from the streamflow data. The use of the maps as predictive tools is restricted largely because of very steep gradients of parameters across the physiography of the study area. The parameters of the larger component are sensitive to over-generalization in the process of creation of the maps. The physical nature of this process changes, as well as the magnitude of the parameters over the study area. While regions of similar values for parameters of the parent distribution can be identified objectively, these regions cannot be used to improve the spatial estimation of the parameters. Within each cluster the physiography and climate are controlling the variability in the parameters in ways that cannot be modelled quantitatively.

The choice of a mixture distribution for the parent of streamflow generating processes is very good, as it has been shown to represent the observed frequency

characteristics of the data, and the parameters of the distribution can be directly interpreted and related to the physical processes active within a basin. This physical reasonableness is a major advantage not shared by the other distributions identified as potential parents. Further improvement in the fit of the model may be obtained with further investigation of the parent distribution and the inclusion of additional processes or the choice of a mixture of different underlying distributions.

CHAPTER 6
FURTHER IMPLICATIONS

A risk estimate, or probability, that a particular ranked discharge will be greater than some value in a given year, rather than just the mean at that rank, is of greater practical value in hydrology. Order statistics provide a method by which the distribution of flows at any rank and the parameters of that distribution may be estimated. The success of the approximation in representing the mean has been demonstrated previously. This chapter presents an extension of the order statistics and the varying success of its application to the flow-duration curves of rivers in two diverse hydrologic environments. Limitations to the practical application of the approach are discussed in light of differing hydrologic processes.

Extensions of the Basic Model

Gibbons (1985) presents the general results of the large-sample approximation to the variance of the r'th order statistics $X_{(r)}$ of a sample n from the continuous distribution F_X. The first and second approximations for both the mean and variance are

$$\text{First approximation: } \sigma^2_{(r)} = g(r)f(F^{-1}(h(r)))^{-2}$$
$$\text{Second approximation: } \sigma^2_{(r)} = g(r)f(F^{-1}(h(r)))^{-2} - (1/4)g^2(r)[f'(F^{-1}(h(r)))f(F^{-1}(h(r)))^{-3}]^2 \quad (20)$$

The approximations are applied to all study locations with very mixed results--so much so that the technique could not be applied universally. Two rivers, the Sumas and Quesnel, which characterize the best and worst hydrologic conditions for the application of the approximation, are shown in detail. The Quesnel is a large basin draining 5,000 km^2 on the western flanks of the Columbia Mountains (gage number 64 in Figure 18). Its regime is dominated by a single snowmelt peak (Figure 28A), while the large basin area and the presence of two large lakes immediately upstream from the gage act to produce highly autocorrelated daily flows (r=0.999). The Sumas (Figure 28B), with a longer high flow season, comprised of many individual storm peaks, has lower a autocorrelation between daily flows (r=0.90).

Figure 29A shows the observed and predicted flow-duration curves for the Sumas River. An improved estimate for the predicted mean may be obtained from a nonlinear regression of the predicted mean against the observed mean and rank

$$\bar{x}'_{(r)} = a\,\bar{x}^{b}_{(r)}\,r^{c} \quad (21)$$

to correct for areas of mis-specification of the curve. The lower Figure (29B) shows the improvement of the corrected form in representing the data, for all ranks; the corrected model curve cannot be distinguished from the observed data. This particular form of regression is ideal for a correction factor because the only possible

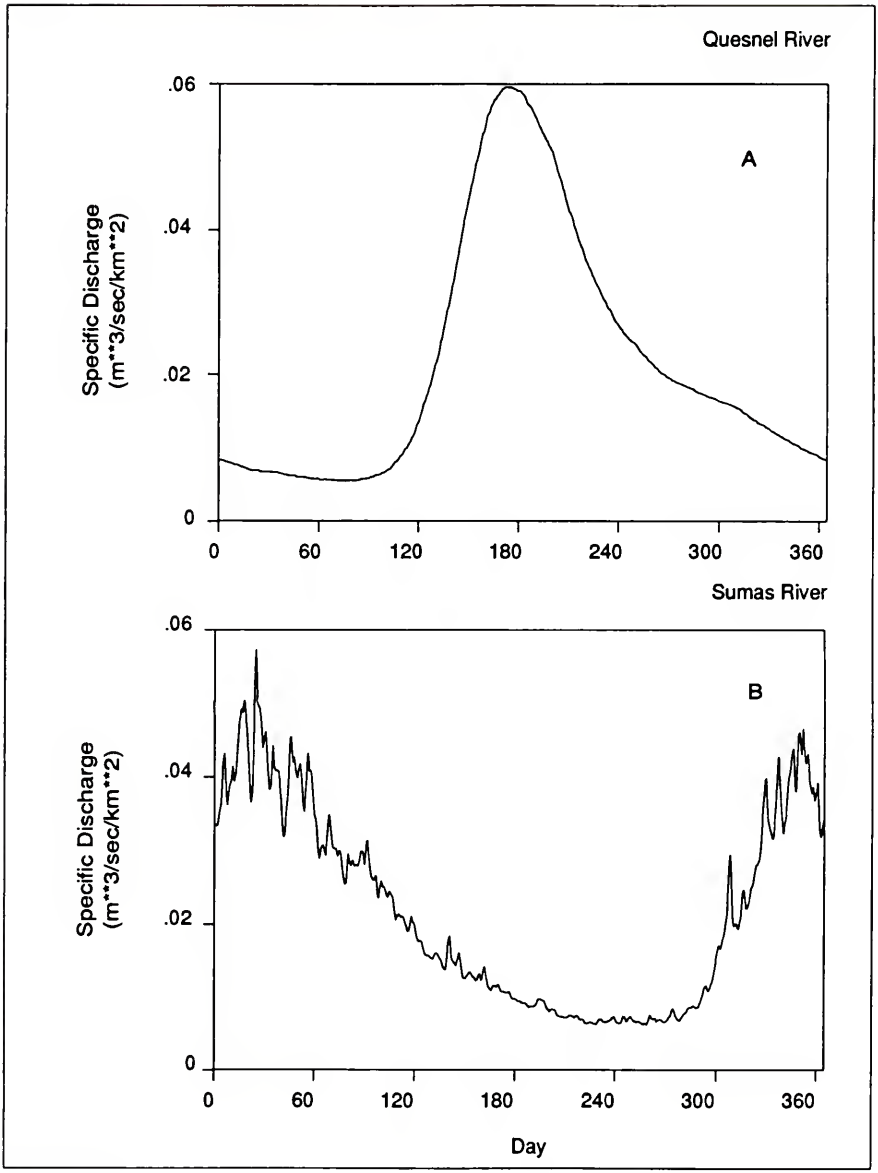

Figure 28. Average Annual Hydrographs. A) Quesnel River; B) Sumas River

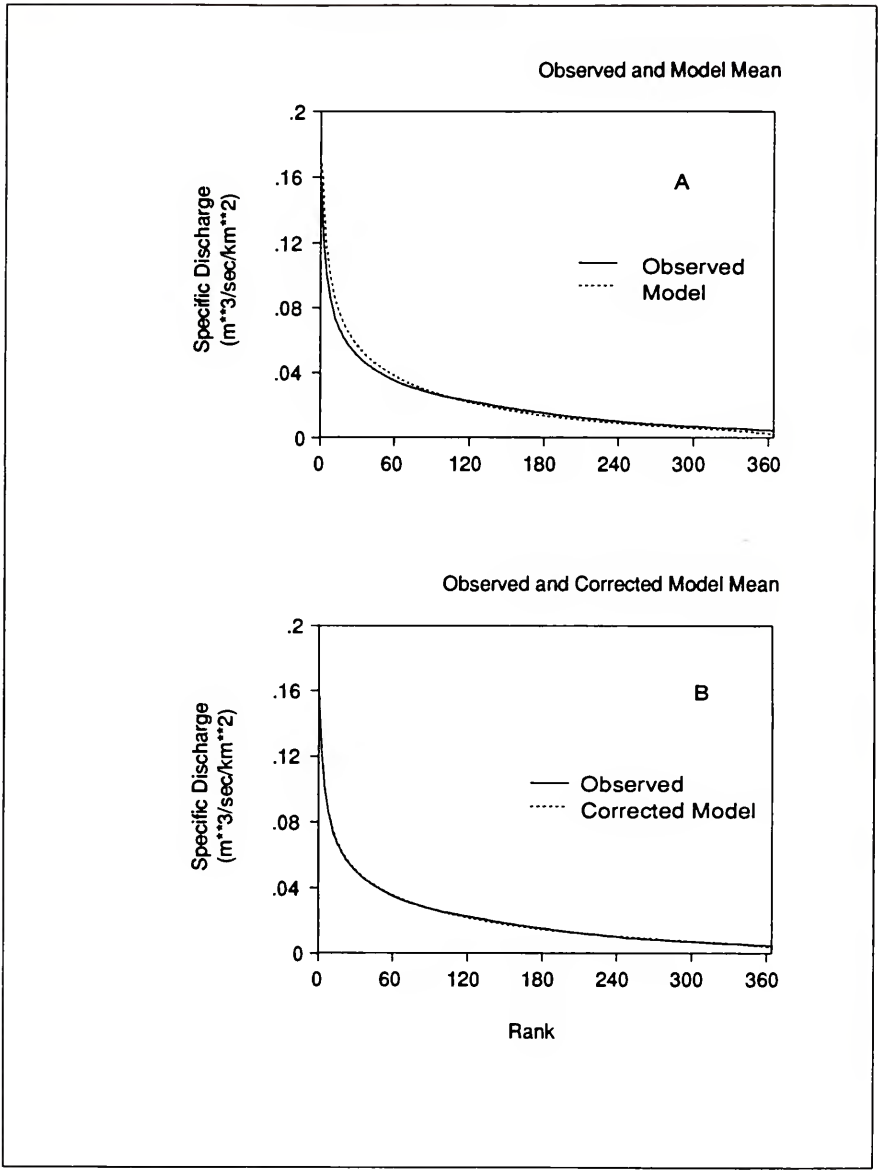

Figure 29. Sumas River, Means by Rank. A) Model; B) Corrected Model

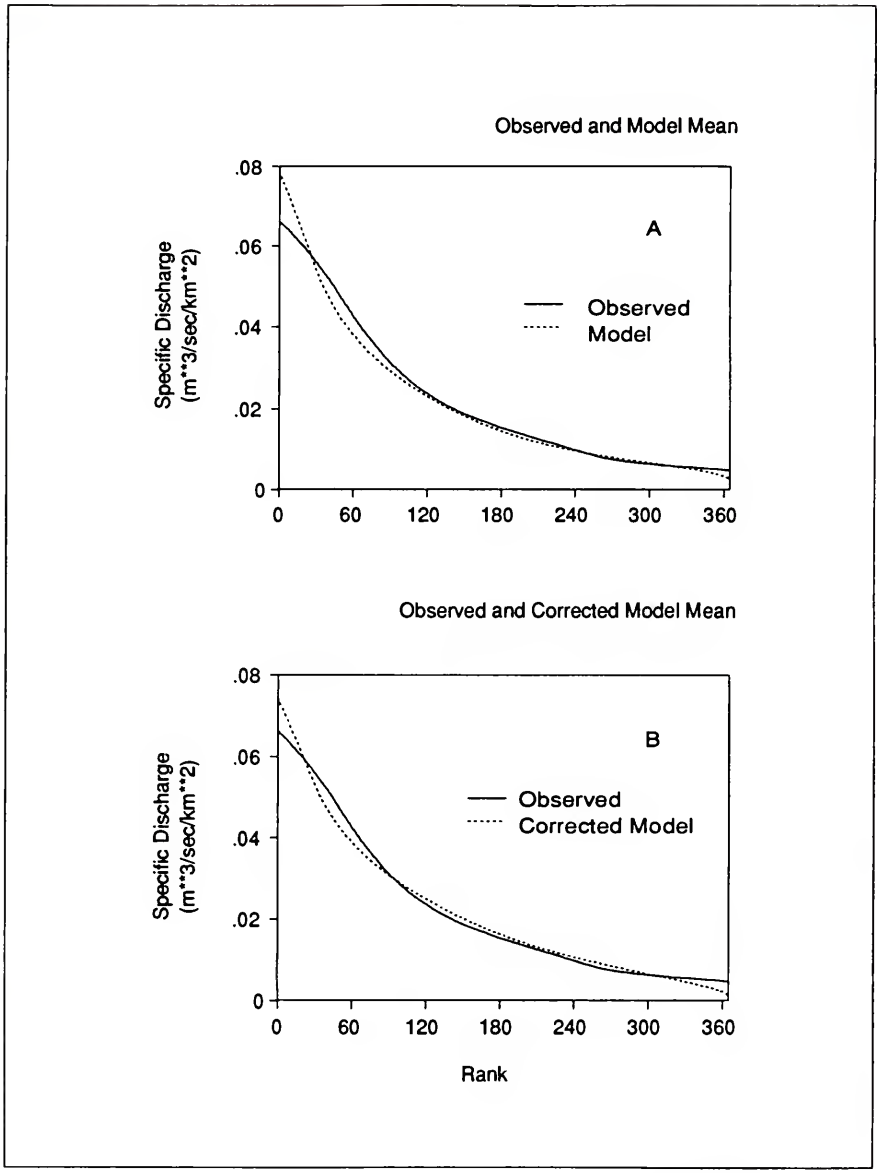

Figure 30. Quesnel River, Means by Rank. A) Model; B) Corrected Model

outcomes of the correction are positive values. A multiple linear regression model was tried, but in some situations the vast majority of small observations at high rank could still be outweighed by very large discharge observations at small rank, giving negative values for the mean at some large ranks.

Table 6. Coefficients of the Regression Correction

	R^2	F	a	b	c
Means:					
Quesnel	98.654	13268.302	0.9176	0.0137	0.9852
Sumas	99.431	31655.563	0.6139	-0.0996	0.7513
Variances:					
Quesnel	99.152	21168.761	0.3033	-0.1448	0.6549
Sumas	98.557	12362.145	0.0265	-0.4667	0.3276

The high storage on the Quesnel River produces a flattening of the upper part of the flow-duration curve. The correction equation is not as successful in reducing the mis-specification of the model (Figure 30B) as in the case of the Sumas. The over-estimation at the lowest ranks is reduced, but the under-estimation at the highest ranks is exaggerated. Table 6 presents a summary of the regression statistics to correct the mean by rank for these two rivers. The R^2 for all the regressions is above 98%; interpretation of this statistic in the untransformed data is difficult as the linear relationships are no longer valid.

Estimates of the variance, by rank, for the Sumas river are shown in Figure 31A. Variances derived from Equation 20 are smaller than those observed for almost all ranks. Application of the non-linear regression correction yields a greatly

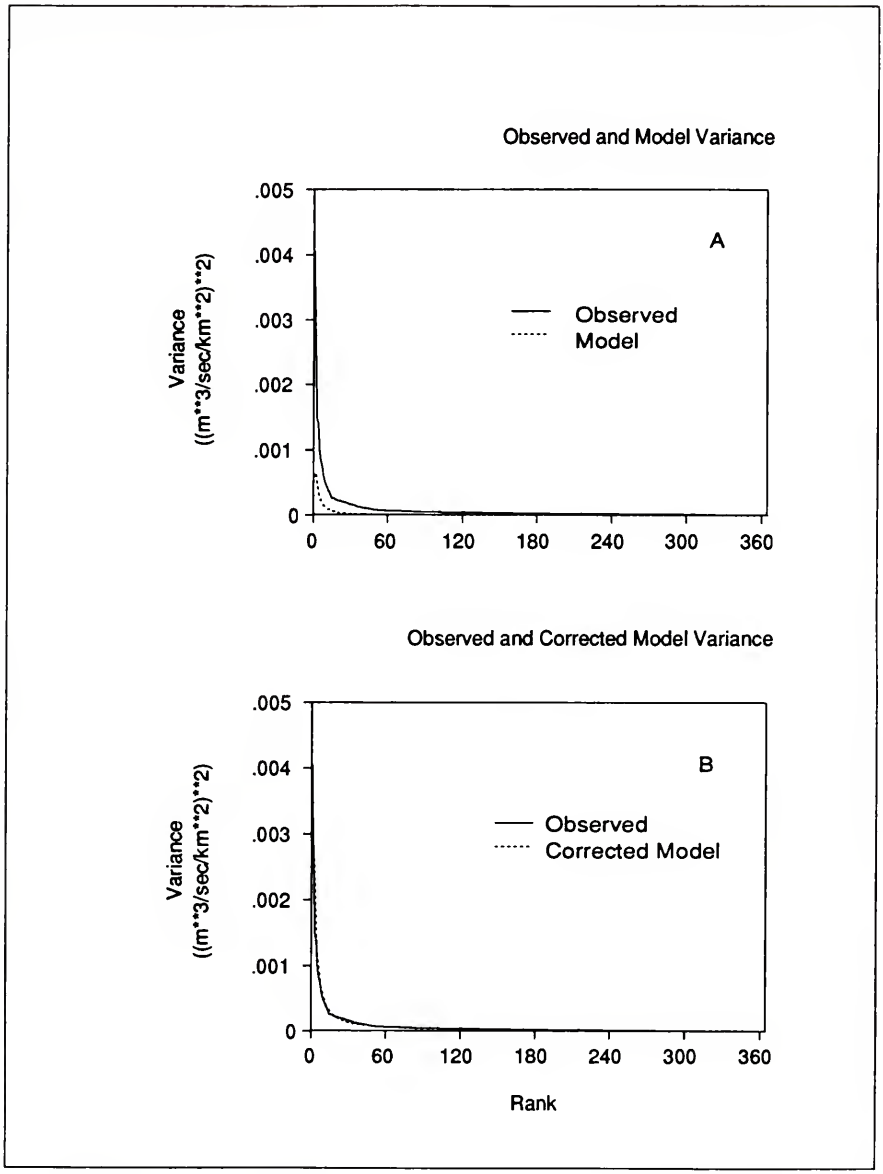

Figure 31. Sumas River, Variances by Rank. A) Model; B) Corrected Model

improved fit on the Sumas (Figure 31B and Table 6) such that at no rank can the observed and corrected variances be said to be significantly different at a level of 0.05 using a standard χ^2 test. In making these tests, it must be assumed that the data for each rank are normally distributed; for the central and intermediate ranks this is theoretically a valid assumption, at extreme ranks this is an inappropriate assumption (Leadbetter et al., 1983).

The variance curves for the Quesnel River (Figure 32 A & B) show a profound mis-identification of the parent distribution. The shape of the predicted curve is wrong and the predicted variances are an order of magnitude too small. While it is possible to inflate the predicted variances using the non-linear regression equation (Figure 32B), the form is still incorrect, particularly at lower ranks where the model regions of the curves do not correspond.

The consistent underestimation of variance on the Sumas River is a function of the inflation of the total variance in the time series resulting from seasonality and daily autocorrelation of streamflows (Figure 28). The situation is exacerbated in the case of the Quesnel (Figure 28A), which is even more markedly seasonal and possesses strong serial correlation ($r=0.999$).

The two rivers display very different characteristics in their respective time series. These differences may be analyzed in terms of two time scales and the physical environments: 1) the daily to monthly patterns of variability, 2) the annual and long term consistency of seasonality, and 3) the different physical characteristics of these basins.

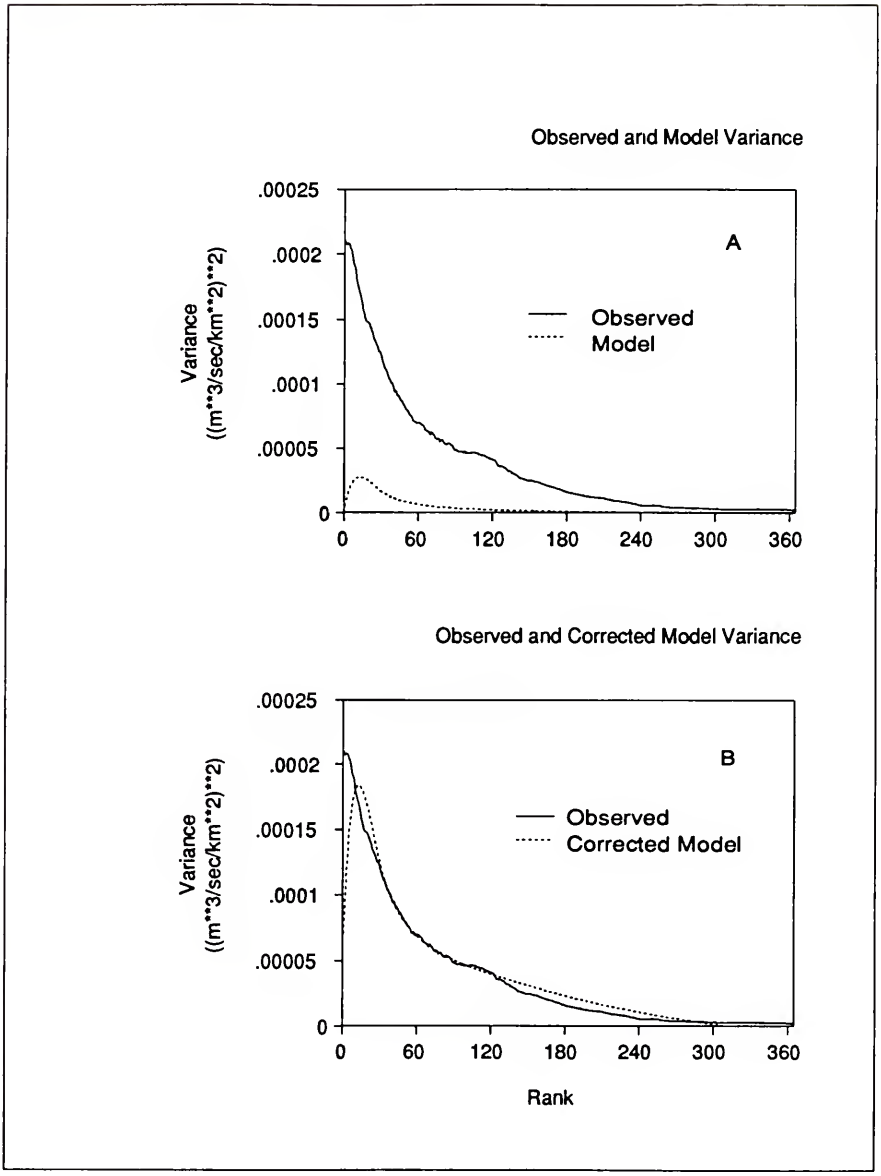

Figure 32. Quesnel River, Variances by Rank. A) Model; B) Corrected Model

On a short term basis, the Sumas and Quesnel rivers differ in the degree of serial correlation; the higher correlation coefficient of the Quesnel River ($r = 0.999$) produces a smoother annual hydrograph than the Sumas ($r = 0.90$). The intrinsic variance in daily flows does not map from the time domain to the frequency domain evenly. The daily variance in the Quesnel River is highest in the spring snowmelt season and very low through the remainder of the year. The Sumas has a longer period of high flow season, but proportionally, the variance is smaller during this period.

The year-to-year consistency of the seasonality function controls variance in the intermediate ranks. For example, the Quesnel has a mean date of the annual flood of day one hundred and fifty, and standard deviation of ten days; the Sumas has a mean date of the annual flood of day ten and a standard deviation of thirty four days (Waylen & Woo, 1983). The more consistent the seasonality function, the higher the variance across all ranks that may be expected.

The large basin area and lake storage on the Quesnel River act in concert to create a large geophysical filter on the daily discharge time series. With a basin area one thirtieth of the Quesnel, the Sumas River shows very little filtering and smoothing of the time series, therefore basin responses are rapid to most inputs. The nature of the streamflow generating processes is also different for each basin. The Sumas River's flows are generated by short duration rainfall events, and their magnitude and frequency are greater in the winter than the summer. The Quesnel

River is dominated by the spring snowmelt, which in turn is controlled by the migration of the 0°C isotherm up the flanks of the Columbia Mountains.

Distributional Results

Once the nature of the parent distribution of daily streamflow is known, the form of the probability distribution of flows at any rank is implied by a combination of order statistics and extreme value theory. The distribution of individual order statistics from an independent and identically distributed parent distribution will belong to one of three forms for extreme orders (Leadbetter et al., 1983), and one of two forms for central and intermediate orders (Wu, 1966). The primary control on the form an extreme order statistic displays is the form of the parent distribution (Leadbetter et al., 1983; Watts et al., 1982). Individual distributions at each set of ranks are examined in more detail below.

All results originate from asymptotic theory and assume sample sizes approaching infinity, but may also hold for large samples. Each of the subsequent limit laws uses notation which refers to a standardized variate X_i; the standardization removes the population characteristics, and leaves a parameter-free variate, similar to a z-score.

Asymptotic Theory of Extreme Order Statistics

An observation belongs to an extreme order if, for any fixed rank k, the limit of rank divided by sample size as sample size approaches infinity, is either 0 or 1

(Leadbetter et al., 1983). When data are ordered in descending order, the limit is 0 for the highest rank, and for the lowest rank the limit is 1. Observations at these ranks may be considered as extreme order statistics.

For an independent and identically distributed population with a known underlying parent distribution, the smaller extremes of the order statistics will follow one of the three extremal forms for minima (David, 1981; Galambos, 1978 & 1984; Leadbetter et al., 1983)

$$P\{\alpha_n(m_n-\beta_n)\le x\}\to H(x)$$

where H is one of the three following extremal types for minima:

$$\text{Type I: } H(x) = 1-\exp(-\exp(x)) \qquad -\infty < x < \infty$$

$$\text{Type II: } H(x) = \begin{cases} 1-\exp(-(-x)^{-\alpha}) & \alpha>0,\ x<0 \\ 1 & x\ge 0 \end{cases}$$

$$\text{Type III: } H(x) = \begin{cases} 0 & x<0 \\ 1-\exp(-x^{\alpha}) & \alpha>0,\ x\ge 0 \end{cases} \qquad (22)$$

Similarly, the larger extremes of the order statistics will follow one of the three extremal forms for maxima (Leadbetter et al., 1983)

$$P\{a_n(M_n-b_n)\le x\}\to G(x)$$

where G is one of the three following extremal types for maxima:

$$\text{Type I: } G(x) = \exp(-\exp(-x)) \qquad -\infty < x < \infty$$

$$\text{Type II: } G(x) = \begin{cases} 0 & x\le 0 \\ \exp(-x^{-\alpha}) & \alpha>0,\ x>0 \end{cases}$$

$$\text{Type III: } G(x) = \begin{cases} \exp(-(-x)^{\alpha}) & \alpha>0,\ x\le 0 \\ 1 & x>0 \end{cases} \qquad (23)$$

The normalizing constants α_n, β_n, a_n and b_n, are derived from the parent distribution. The forms of these constants have been derived for most distributions and are detailed in Table 7 (Leadbetter et al., 1983; Takahashi, 1987).

Independent and identically distributed populations are assumed throughout. However it has been shown that this may be relaxed and that the model is robust at least for a Gaussian process with regards to dependence and stationarity (Kinnison, 1985). Loynes (1965) reports that the three limit laws for the extremes are the same for the independent and dependent cases. Berman (1962) has also shown that the assumption of an identical parent distribution may be relaxed and the limits will still hold.

Asymptotic Theory of Central and Intermediate Order Statistics

An observation belongs to a central or intermediate order if, for any fixed rank k, the limit of rank divided by sample size, as sample size approaches infinity, is a constant, and not equal to either 0 or 1 (Leadbetter et al., 1983).

For an independent and identically distributed population the central order statistics will follow the normal distribution (Hoeffding, 1948; Mood et al., 1963)

$$P\{a_n(M_n^{(k_n)}-b_n)\le x\}\to H(x)$$

where H is one of the four following types for the k^{th} central rank:

$$H(x) = \begin{cases} 0 & x<0 \\ \Phi(cx^{\alpha}) & x\ge 0, c>0, \alpha>0 \end{cases}$$

$$H(x) = \begin{cases} \Phi(-c|x|^{\alpha}) & x<0, c>0, \alpha>0 \\ 1 & x\ge 0 \end{cases} \qquad (24)$$

$$H(x) = \begin{cases} \Phi(-c_1|x|^{\alpha}) & x<0, c_1>0, \alpha>0 \\ \Phi(c_2x^{\alpha}) & x\ge 0, c_2>0, \alpha>0 \end{cases}$$

$$H(x) = \begin{cases} 0 & x<-1 \\ 1/2 & -1\le x<1 \\ 1 & x\ge 1 \end{cases}$$

The intermediate order statistics will follow either the normal or lognormal distribution (Chibisov, 1964; Leadbetter et al., 1983; Wu, 1966)

$$P\{a_n(M_n^{(k_n)}-b_n)\le x\}\to H(x)$$

where H is one of the three following types for the k^{th} intermediate rank:

$$H(x) = \begin{cases} \Phi(-a\log|x|) & x<0, a>0 \\ 1 & x\ge 0, \end{cases} \qquad (25)$$

$$H(x) = \begin{cases} 0 & x\le 0 \\ \Phi(a\log x) & x>0, a>0 \end{cases}$$

$$H(x) = \Phi(x) \qquad -\infty<x<\infty$$

where $\Phi(x)$ is a standard normal distribution and $\alpha > 0$. The constants a_n and b_n are distribution dependent; for a central rank, they are derived from the distribution function of the parent (Bahadur, 1966; Kendall & Stuart, 1977). For an intermediate rank, the normalizing constants are distributionally dependent also (Watts, 1980).

These results are robust with regards to stationarity and independence for a Gaussian parent (Watts, 1980; Watts et al. 1982).

Table 8. Domains of Attraction by Type

Distribution Type (Parent)	for Maxima	for Minima
Normal	Type I	Type I
Exponential	Type I	Type III
Lognormal	Type I	Type I
Gamma	Type I	Type III
Gumbel (Type I), maxima	Type I	Type I
Gumbel (Type I), minima	Type I	Type I
Frechet (Type II), maxima	Type II	Type I
Frechet (Type II), minima	Type I	Type II
Weibull (Type III), maxima	Type III	Type I
Weibull (Type III), minima	Type I	Type III
GEV Type I, $k=0$, maxima	Type I	Type I
GEV Type II, $k<0$, maxima	Type II	Type I
GEV Type III, $k>0$, maxima	Type III	Type I

Reference: Leadbetter et al., 1983

The parent distribution may belong to one of three domains of attraction for extreme order statistics, and one of two for central and intermediate orders. The criteria for selecting which extreme value domain of attraction a particular parent distribution belongs to has been described in Leadbetter et al. (1983) and is listed in Table 8. Many of the distributions used commonly in hydrology to model daily discharge (as a stochastic residual in a deterministic model) belong to the type I domain. The normal, lognormal, gamma and exponential distributions all limit in the extreme orders to a type I distribution (Leadbetter et al., 1983; Takahashi, 1987). Each of the three extreme value distributions limits to its own distributional form in

Table 7. Normalizing Constants for Maxima

Type I domain	a_n	b_n
Normal	$(2\log n)^{1/2}$	$(2\log n)^{1/2}-1/2(2\log n)^{-1/2}$ $(\log\log n + \log 4\pi)$
Exponential	1	$\log n$
Log-normal	$a_n e^{-bn}$	e^{bn}, where a_n and b_n from normal
Gamma	1	$\log n + (\eta-1)\log\log n - \log \Gamma(\eta)$
Gumbel (Type I)	1	$\log n$
GEV, $k=0$	1	$\log n$
Type II domain	a_n	b_n
Frechet (Type II)	$n^{-1/\alpha}$	0
GEV, $k<0$	$n^{1/\alpha}$	0
Type III domain	a_n	b_n
Weibull (Type III)	$n^{1/\alpha}$	0
GEV, $k>0$	$n^{1/\alpha}$	0

References: Leadbetter et al., 1983; Takahashi, 1987

the extreme (Leadbetter et al., 1983), thus the generalized extreme value distribution will have three possible limiting forms for the extremes.

The domain of attraction for central ranks is the normal distribution. The distribution of the median, and other central orders will have a normal or Gaussian form (Bahadur, 1966; Galambos, 1984; Mood et al., 1963; Sarhan & Greenberg, 1962). Chibisov (1964) has detailed the distributional requirements for each of the domains of attraction. The domains of attraction for intermediate ranks are less clearly defined. Order statistics may have one of two distributional forms, the normal or lognormal (Balkema et al., 1978a & 1978b; Chibisov, 1964; Wu, 1966). Despite the relatively slow rates of convergence to the asymptotic forms associated with normal or lognormal parents (Leadbetter et al., 1983; David, 1981), use of asymptotic theory allows the distribution for any rank to be specified on the basis of the parent distribution and it's parameters. In the case of a mixture parent distribution, the order statistic-based model is robust (Loynes (1965); Berman (1962)). Under these conditions, the normalizing constants are determined by the sub-population in the direction of interest (e.g. the smaller sub-population for minima, and the larger sub-population for maxima).

Figure 33 shows the distribution of the annual flood (rank 1), the annual median (rank 182), and the annual low-flow (rank 365) for the Sumas River. Parameters have been estimated using the method of moments estimators, from the corrected means and variances derived from application of order statistics to the mixture lognormal parent distribution. The normalizing constants are not derived

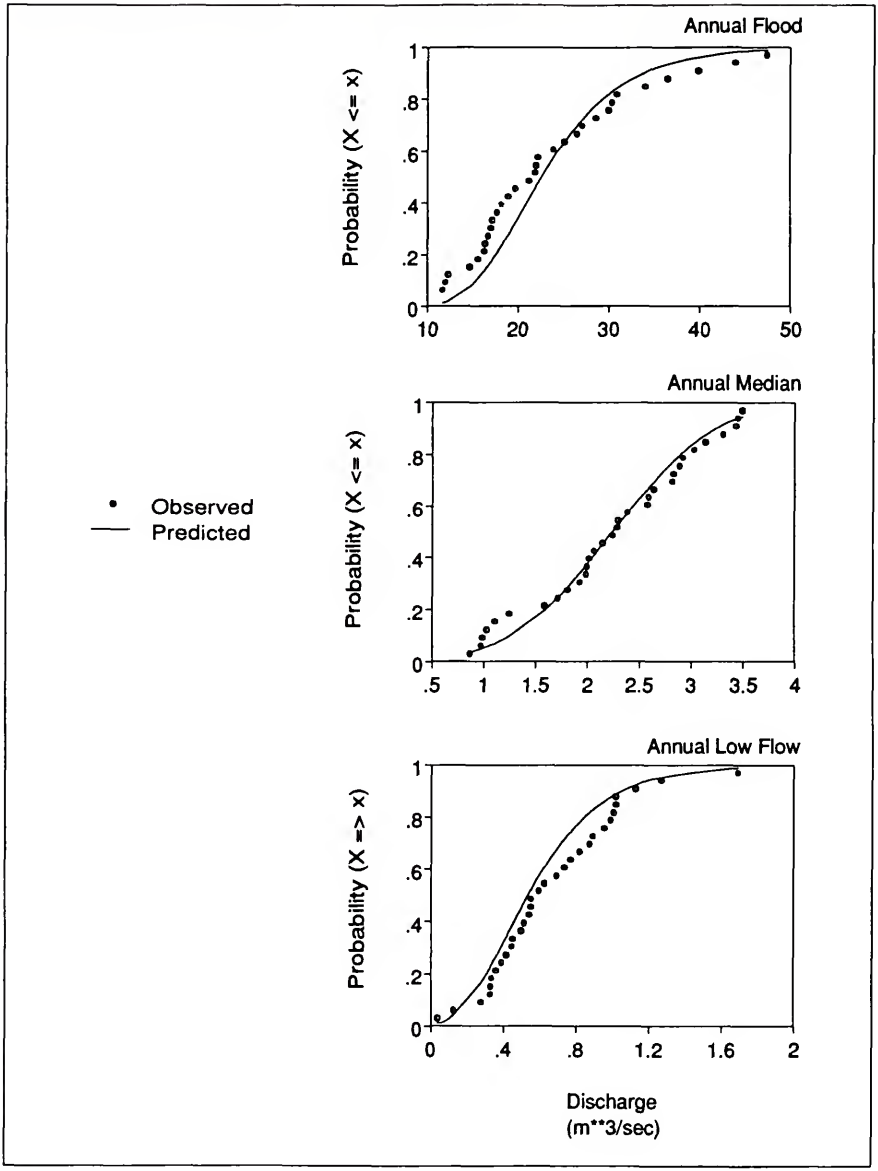

Figure 33. Sumas River, Annual Distributions

directly from the parent distribution, as the severe underestimation of the variance of all ranks is carried through to the estimation in the asymptotic results. These results are used to predict the shape of the probability distribution for the particular ranks: the type I extreme value distribution for both the annual flood (maxima) and low-flows (minima), and the normal distribution for the annual median. In all cases the predicted distributions cannot be considered significantly different from the observed using the K-S test with a significance level of 0.20 (Table 9).

Concluding Remarks

Identification of the parent distribution of daily discharge allows the subsequent estimation of rank means and variances for the flow-duration curve. The inclusion of the estimation of variance by rank, adds information about the structure and reliability of the flow-duration curve. Both sets of information may be derived from the form and parameters of the parent distribution, instead of the calculation of each from the ranked observations of daily discharge. Figure 34 presents the mean plus and minus one standard deviation by rank. Both the observed and corrected model forms are included in this graph. There is little observable difference between the two sets of curves.

Asymptotic theory may be used to predict the shape of the probability distribution of discharge at any rank based on identification of the parent distribution alone. Using the first two moments for ranked data generated from the large sample approximations, derived from the parent distribution, in turn, permits the estimation

Table 9. Sumas River, Annual Distribution Parameters

Distribution		Mean	Standard deviation	α	β
Annual Flood	Observed	23.303	9.341	7.28	19.1
	Predicted	23.819	7.628	5.945	20.387
	$\alpha = s_x/1.283$ $\beta = x - 0.45s_x$				
	Type I distribution for maxima K-S Statistic for predicted parameters				0.1581
Annual Median	Observed	2.246	0.77		
	Predicted	2.011	0.76		
	Normal distribution K-S Statistic for predicted parameters				0.0833
Annual Low-flow	Observed	0.665	0.35	0.273	0.822
	Predicted	0.279	0.35	0.273	0.437
	$\alpha = s_x/1.283$ $\beta = x + 0.45s_x$				
	Type I distribution for minima K-S Statistic for predicted parameters				0.1212

Reference: Haan, 1977

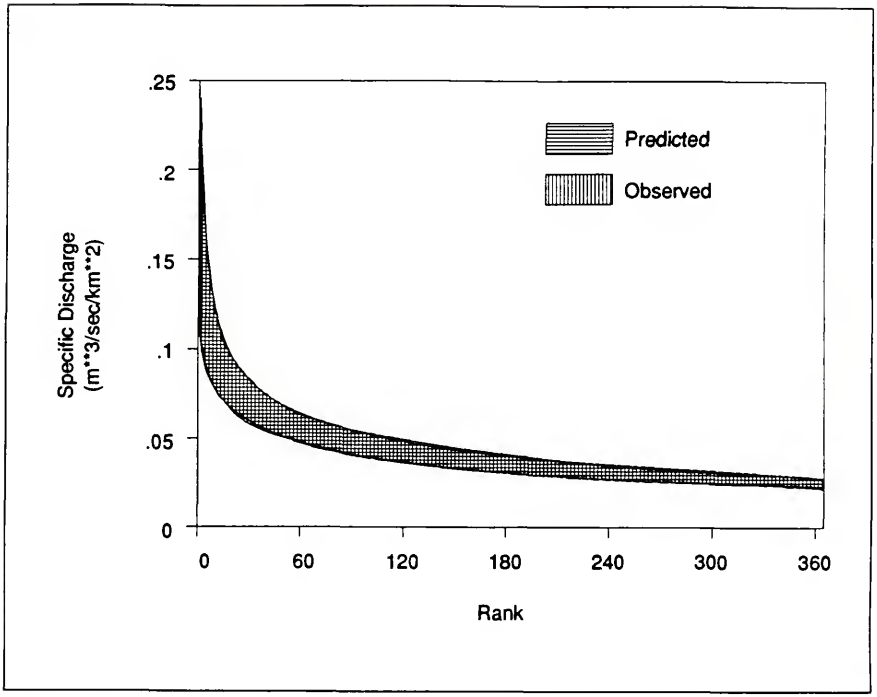

Figure 34. Sumas River, Flow-Duration Curve with Standard Deviations

of the parameters of the rank distribution. These two results, the parameters and the form of the distribution, by rank, completely summarize the flow-frequency regime of a river.

Identification of the parent distribution is the critical step in achieving these results. The mixture distribution has been shown to be a flexible parent distribution in the highly seasonal regimes of British Columbia. Performance of the mixture distribution in the prediction of the rank mean may be improved through the use of a non-linear regression equation. The prediction of variance, by rank is more difficult; the effects of seasonality and basin characteristics manifest themselves in patterns of variability in both the time and frequency domains. Both seasonality and basin area have the effect of inflating the variances at middle ranks. Lake storage tends to reduce or level the variance at the lowest ranks.

Estimation of higher order moments, such as the variance, are susceptible to outliers, especially with restricted sample sizes. In general the higher order moments are very unreliable (have a high variance) for small sample sizes. For example, the variance of the sample variance is $[M_4 - (n-3)s^4/(n-1)]/n$, where M_4 is the fourth moment about the mean, s is the standard deviation and n is the sample size (Haan, 1977). The length of available record for the Sumas River is 32 years and for the Quesnel River 52, which provide 11,680 and 18,980 daily observations, respectively, for estimation of the parent distribution. However, in estimating the moments at a given rank, sample sizes are reduced to 32 and 52 again.

The mixture model for the parent distribution is adequate for the description of many streamflow regimes, however, the effects of some geophysical factors such as seasonality, basin area and storage act as bounds on the distribution of streamflow. The parent distribution should reflect these upper and lower bounds and the serial correlations which may be present. In the cases where the mixture distribution has proved to be inadequate, further investigation is warranted to identify the underlying frequency structures for daily discharge.

CHAPTER 7
CONCLUSIONS

Recently there has been considerable concern about the correct marriage of statistical methods and physical understanding of the streamflow generating processes (Klemeš, 1986; Potter, 1987). This dissertation addresses the applied problem of the representation and prediction of flow-duration curves in a way that provides a strong theoretical basis of analysis and preserves the physical underpinnings of the hydrologic processes, thereby enhancing the soundness of the results. This is achieved through the correct application of concepts in physical geography, hydrology and statistics, which provide the foundation for the research approach. More specifically, this encompasses seven major components: 1) the foundation of the research upon the physical geography and climate of the region, 2) the use of the basic geomorphic unit of the drainage basin, 3) development of the model at basin scale and leading to regional scales, 4) the unification of a mixed mechanism for the generation of flows and its statistical analog, 5) identification of statistically and hydrologically homogeneous regions, 6) prediction of the parameters of the model for ungaged basins, and 7) parsimony.

The final model which is numerically based in large-sample approximations of order statistics, duplicates the otherwise empirical construction technique of the calendar-year flow-duration curve. Moreover the use of a mixed lognormal

distribution as the parent permits the representation of a variety of flow regimes, but involves no loss of analytical power, as the necessary solutions are well documented. The model is relatively simple to use, requiring the straight forward estimation of five parameters to summarize the three hundred and sixty five calculated means of the flow-duration curve. Ultimately these may be interpreted physically and correspond closely to the average duration and magnitude of the streamflow generating processes in a basin.

The spatial distribution of the parent distribution's parameters permit the geographic interpretation of the structure of the stochastic model. Each show variations consistent with the known physiographic and hydrometeorologic gradients in the province of British Columbia. The resultant maps correspond directly to the general climate and physical variables which control streamflow generation in the study area. Seven reasonably homogeneous sub-regions can be objectively identified within the Province, each of which corresponds to a hydrologically distinct environment, both in terms of available hydrometeorological data and the spatially limited existing studies of other streamflow variables. Identification of both reasonably homogeneous regions and the gradients of the parameters across those regions facilitates spatial interpolation and the prediction of flow-duration curves for rivers where streamflow records are incomplete or unavailable.

The application of the model over a wide variety of hydrologic environments demonstrates the robustness of both the overall technique and to a lesser extent the selection of the mixed lognormal distribution as a parent. The basic modelling

procedure may be used in any hydrologic environment. However, relative performance of the parent appears to vary with the local conditions. In particular, small, low storage, rainfall-dominated regimes seem to be particularly suited to this analysis. This is particularly noticeable when the technique is applied to the estimation of the risk associated with each ranked discharge as opposed to solely the mean at that rank. The problem may be reduced in the future by selection of alternate parent distributions, although none of the standard distributions used to represent streamflow performed as well as the mixed lognormal in this study. The use of additional statistical sub-populations may be the most readily accomplished means of improving the model. Both the number of sub-populations and the form each takes may be varied. This may provide upper and lower bounds for streamflow and intermittent processes. However, the addition of sub-populations requires the a priori identification and subdivision of the data by process. Alternately, a less physically based distribution such as the Wakeby may be employed, but the theoretical development of this distribution is incomplete. The limitations of the statistical theory in including highly seasonal and autocorrelated data restricts the future development of the model.

The use of maps for spatial estimation and prediction at ungaged locations is presently restricted to areas with nearby gaged basins in similar environments. The steep hydrologic gradients in many mountainous parts of the study area limit the accuracy of interpolation. The numerical techniques employed to represent the

spatial distributions of parameters are inadequate to handle the observed gradients and the sparsity of data in some areas of the province.

In summary, a flexible and convenient probabilistic modelling technique has been developed for representation of the calendar-year flow-duration curve. The model provides a way of summarizing the flow-duration curve with a restricted number of parameters, as well as providing a theoretical justification for its construction. The inclusion of variance within the model offers great potential for representing the frequency characteristics of a river in more analytical depth. With the addition of distributional forms for each rank, the entire flow-frequency regime of a river may be summarized and represented by a few parameters. This in turn can be used to change a largely descriptive tool of hydrologists into an analytical tool for the analysis of risk in streamflow studies. The model presented maintains strong links between statistical theory and both physical and operational hydrology. This ensures a deeper understanding of the statistical processes which control the common flood measures, and in turn, this knowledge of the parent distribution may be used to strengthen these same flow-frequency measures.

APPENDIX A
LISTING OF GAGING STATIONS

Map Key	Station Identifier	Station Name / Latitude	Longitude	Basin Area	Record
1	07EA001	Finlay River at Ware			
		57-25-15"N	125-37-35"W	11,100 km^2	20 years
2	07EA002	Kwadacha River near Ware			
		57-27-00	125-38-15	2,720	22
3	07ED001	Nation River near Fort St James			
		55-12-00	124-14-00	4,350	20
4	07EE007	Parsnip River above Misinchinka River			
		55-04-41	122-54-17	4,900	20
5	07FA001	Halfway River near Farrel Creek (Lower Station)			
		56-13-40	121-28-50	9,400	19
6	07FB001	Pine River at East Pine			
		55-43-12	121-12-28	12,100	23
7	07FC001	Beatton River near Fort St John			
		56-16-12	120-40-43	15,600	22
8	07FD001	Kiskatinaw River near Farmington			
		55-57-25	120-22-50	3,570	24
9	08CB001	Stikine River above Grand Canyon			
		58-02-38	129-56-45	18,800	23
10	08CD001	Tuya River near Telegraph Creek			
		58-04-20	130-49-27	3,600	21
11	08CE001	Stikine River at Telegraph Creek			
		57-54-03	131-09-16	29,300	23
12	08CG001	Iskut River below Johnson River			
		56-44-20	131-40-25	9,350	23
13	08CG003	Iskut River at Outlet of Kinaskan Lake			
		57-32-00	130-12-28	1,250	21
14	08CG004	Iskut River above Snippaker Creek			
		56-41-55	130-52-23	7,230	19
15	08DA005	Surprise Creek near the Mouth			
		56-06-35	129-28-33	221	19
16	08DB001	Nass River above Shumal Creek			
		55-12-50	129-08-20	19,200	34

17	08DC006	Bear River above Bitter Creek			
		56-02-34	129-55-30	350	20
18	08DD001	Unuk River near Stewart			
		56-21-05	130-41-30	1,480	21
19	08EB003	Skeena River at Glen Vowell			
		55-18-09	127-40-11	25,900	25
20	08EB004	Kispiox River near Hazelton			
		55-28-00	127-44-31	1,870	21
21	08EC001	Babine River at Babine			
		55-19-00	126-37-30	6,480	39
22	08ED002	Morice River near Houston			
		54-07-05	127-25-26	1,910	26
23	08EE004	Bulkley River at Quick			
		54-37-05	126-53-55	7,360	40
24	08EE008	Goathorn Creek near Telkwa			
		54-38-50	127-07-20	132	24
25	08EF001	Skeena River at Usk			
		54-37-50	128-25-40	42,200	45
26	08EF005	Zymoetz River above O.K. Creek			
		54-29-00	128-19-50	2,980	23
27	08EG011	Zymagotitz River near Terrace			
		54-31-07	128-43-40	376	25
28	08EG012	Exchamsiks River near Terrace			
		54-21-47	129-18-41	370	23
29	08FA002	Wannock River at Outlet of Owikeno Lake			
		51-40-40	127-10-30	3,940	29
30	08FB002	Bella Coola River near Hagensborg			
		52-26-32	126-19-56	4,040	18
31	08FB007	Bella Coola River above Burnt Bridge Creek			
		52-25-20	126-09-29	3,730	17
32	08FF001	Kitimat River below Hirsch Creek			
		54-03-34	128-36-00	1,990	21
33	08FF002	Hirsch Creek near the Mouth			
		54-03-48	128-36-00	347	21
34	08FF003	Little Wedeene River below Bowbyes Creek			
		54-08-11	128-41-24	188	19
35	08GA024	Cheakamus River near Mons			
		50-05-00	123-02-54	287	23
36	08GD004	Homathko River at the Mouth			
		50-59-05	124-55-01	5,720	19
37	08HA001	Chemainus River near Westholme			
		48-52-45	123-42-07	355	33
38	08HA003	Koksilah River at Cowichan Station			
		48-43-39	123-40-11	209	30

39	08HA010	San Juan River near Port Renfrew			
		48-34-38	124-17-49	580	23
40	08HA016	Bings Creek near the Mouth			
		48-47-22	123-43-28	15.50	20
41	08HB004	Little Qualicum River at Outlet of Cameron Lake			
		49-17-27	124-35-00	135	34
42	08HB014	Sarita River near Bamfield			
		48-53-34	124-57-54	162	34
43	08HB024	Tsable River near Fanny Bay			
		49-31-03	124-50-30	113	26
44	08HB029	Little Qualicum River near Qualicum Beach			
		49-21-07	124-29-07	237	24
45	08HD001	Campbell River at Outlet of Campbell Lake			
		50-00-08	125-23-20	1,400	37
46	08HE006	Zeballos River near Zeballos			
		50-00-52	126-50-33	181	23
47	08JA002	Ootsa River at Ootsa Lake			
		53-37-30	125-44-00	4,450	18
48	08JB002	Stellako River at Glenannan			
		54-00-38	125-00-01	3,600	37
49	08JE001	Stuart River near Fort St James			
		54-25-05	124-16-30	14,600	45
50	08KA001	Dore River near McBride			
		53-18-38	120-14-45	404	21
51	08KA004	Fraser River at Hansard			
		54-04-43	121-50-52	18,000	34
52	08KA005	Fraser River at McBride			
		53-17-10	120-06-46	6,890	30
53	08KA007	Fraser River at Red Pass			
		52-58-55	119-00-15	1,700	31
54	08KA008	Moose River near Red Pass			
		52-55-12	118-48-00	458	29
55	08KA009	McKale River near 940m Contour			
		53-26-41	120-13-10	252	16
56	08KB001	Fraser River at Shelley			
		54-00-40	122-37-00	32,400	37
57	08KB003	McGregor River at Lower Canyon			
		54-13-56	121-40-00	4,770	27
58	08KC001	Salmon River near Prince George			
		54-05-46	122-40-41	4,300	28
59	08KD001	Bowron River near Wells			
		53-15-40	121-24-50	458	24
60	08KD003	Willow River near Willow River			
		54-04-07	122-27-50	3,110	22

61	08KE009	Cottonwood River near Cinema			
		53-09-15	122-28-30	1,910	22
62	08KF001	Nazko River above Michelle Creek			
		52-55-45	123-32-50	3,240	20
63	08KG001	West Road River near Cinema			
		53-18-40	122-53-15	12,400	17
64	08KH001	Quesnel River at Likely			
		52-36-54	121-34-15	5,930	52
65	08KH003	Cariboo River below Kangaroo Creek			
		52-40-00	121-40-07	3,260	48
66	08KH006	Quesnel River near Quesnel			
		52-50-37	122-13-25	11,500	41
67	08KH010	Horsefly River above McKinley Creek			
		52-17-23	121-03-37	785	25
68	08KH014	Mitchell River at Outlet of Mitchell Lake			
		52-50-36	120-44-30	245	17
69	08KH019	Moffat Creek near Horsefly			
		52-18-49	121-24-10	539	21
70	08LA001	Clearwater River near Clearwater Station			
		51-39-20	120-03-55	10,200	45
71	08LA004	Murtle River above Dawson Falls			
		51-58-46	120-06-13	1,380	28
72	08LA007	Clearwater River at Outlet of Clearwater Lake			
		52-08-20	120-11-30	2,950	29
73	08LA008	Mahood River at Outlet of Mahood Lake			
		51-55-55	120-14-29	4,710	22
74	08LA013	Clearwater River at Outlet of Hobson Lake			
		52-25-18	120-19-26	904	23
75	08LB020	Barriere River at the Mouth			
		51-10-48	120-07-50	1,140	36
76	08LB022	North Thompson River near Barriere			
		51-11-10	120-09-05	17,700	39
77	08LB047	North Thompson River at Birch Island			
		51-36-10	119-54-55	4,450	27
78	08LB064	North Thompson River at McLure			
		51-02-29	120-14-28	19,600	29
79	08LB069	Barriere River below Sprague Creek			
		51-14-50	119-55-50	624	22
80	08LD001	Adams River near Squilax			
		50-57-14	119-40-25	3,080	57
81	08LE024	Eagle River near Mulakwa			
		50-56-15	118-47-59	904	22
82	08LE027	Seymour River near Seymour Arm			
		51-15-45	118-56-48	805	18

83	08LE031	South Thompson River at Chase			
		50-49-37	119-41-57	16,200	58
84	08LF022	Thompson River at Spences Bridge			
		50-25-15	121-20-30	54,600	36
85	08LG048	Coldwater River near Brookmere			
		49-51-20	120-54-27	316	20
86	08LG056	Guichon Creek above Tunkwa Lake Diversion			
		50-36-30	120-54-39	78.20	19
87	08MA001	Chilko River near Redstone			
		52-04-12	123-32-12	6,940	35
88	08MA002	Chilko River at Outlet of Chilko Lake			
		51-37-31	124-08-31	2,110	26
89	08ME004	Bridge River at Lajoie Falls			
		50-50-15	122-51-24	956	21
90	08MF003	Coquihalla River near Hope			
		49-22-31	121-25-10	741	32
91	08MF062	Coquihalla River below Needle Creek			
		49-33-33	121-06-20	85.50	20
92	08MG003	Green River near Pemberton			
		50-16-55	122-51-05	855	34
93	08MG004	Green River near Rainbow			
		50-10-15	122-55-00	195	26
94	08MG005	Lillooet River near Pemberton			
		50-20-08	122-47-58	2,160	61
95	08MG006	Rutherford Creek near Pemberton			
		50-16-00	122-52-20	179	23
96	08MG007	Soo River near Pemberton			
		50-13-30	122-53-00	283	23
97	08MG013	Harrison River near Harrison Hot Springs			
		49-18-40	121-48-08	7,870	35
98	08MH001	Chilliwack River at Vedder Crossing			
		49-05-50	121-57-45	1,230	51
99	08MH006	North Alouette River at 232nd Street, Maple Ridge			
		49-14-34	122-34-42	37.30	28
100	08MH016	Chilliwack River at Outlet of Chilliwack Lake			
		49-05-02	121-27-24	329	54
101	08MH029	Sumas River near Huntingdon			
		49-00-09	122-13-50	149	32
102	08MH056	Slesse Creek near Vedder Crossing			
		49-04-16	121-41-58	162	24
103	08MH076	Kanaka Creek near Webster Corners			
		49-12-25	122-32-05	47.70	25
104	08MH103	Chilliwack River above Slesse Creek			
		49-06-09	121-39-42	645	24

105	08MH104	Anderson Creek at the Mouth			
		49-05-34	122-40-58	27.20	21
106	08MH105	Nicomekl River below Murray Creek			
		49-06-02	122-38-44	64.50	18
107	08NA002	Columbia River at Nicholson			
		51-14-38	116-54-42	6,660	68
108	08NA024	Windermere Creek near Windermere			
		50-29-03	115-56-02	84.20	20
109	08NA045	Columbia River near Fairmont Hot Springs			
		50-19-25	115-51-45	891	42
110	08ND006	Columbia River at Twelve Mile Ferry			
		50-51-11	118-06-24	28,500	19
111	08ND009	Downie Creek near Revelstoke			
		51-28-37	118-25-32	655	23
112	08ND012	Goldstream River below Old Camp Creek			
		51-40-06	118-35-49	938	23
113	08ND013	Illecillewaet River at Greeley			
		51-00-49	118-04-57	1,170	24
114	08ND014	Jordan River above Kirkup Creek			
		51-02-40	118-16-09	272	24
115	08NE001	Incomappleux River near Beaton			
		50-46-25	117-40-36	1,020	35
116	08NE006	Kuskanax Creek near Nakusp			
		50-15-03	117-48-44	350	24
117	08NE008	Beaton Creek near Beaton			
		50-44-09	117-43-44	99.50	33
118	08NE039	Big Sheep Creek near Rossland			
		49-01-00	117-56-40	347	38
119	08NE074	Salmo River near Salmo			
		49-04-07	117-16-37	1,230	38
120	08NE077	Barnes Creek near Needles			
		49-54-27	118-07-31	201	37
121	08NE087	Deer Creek at Deer Park			
		49-25-30	118-02-30	80.50	27
122	08NF002	Kootenay River at Canal Flats			
		50-08-52	115-47-57	5,390	29
123	08NG005	Kootenay River at Wardner			
		49-25-13	115-25-10	13,600	54
124	08NG012	St Mary River at Wycliffe			
		49-36-01	115-51-46	2,360	38
125	08NG042	Kootenay River at Newgate			
		49-00-52	115-10-27	20,000	41
126	08NG046	St Mary River near Marysville			
		49-36-29	116-10-07	1,480	40

127	08NG051	Skookumchuck Creek near Skookumchuck			
		49-54-42	115-46-03	637	26
128	08NG053	Kootenay River near Skookumchuck			
		49-54-38	115-44-08	7,120	37
129	08NG065	Kootenay River at Fort Steele			
		49-36-43	115-38-04	11,400	24
130	08NH001	Duncan River near Howser			
		50-17-09	116-56-44	2,160	36
131	08NH005	Kaslo River below Kemp Creek			
		49-54-27	116-57-07	453	27
132	08NH006	Moyie River at Eastport			
		48-59-58	116-10-43	1,480	58
133	08NH007	Lardeau River at Marblehead			
		50-15-47	116-58-02	1,620	43
134	08NH032	Boundary Creek near Porthill			
		48-59-50	116-34-05	251	57
135	08NH034	Moyie River at Moyie			
		49-15-30	115-51-00	725	23
136	08NH066	Lardeau River at Gerrard			
		50-30-36	117-16-29	769	20
137	08NH084	Arrow Creek near Erickson			
		49-09-32	116-27-04	78.70	24
138	08NH115	Sullivan Creek near Canyon			
		49-06-14	116-25-35	6.22	21
139	08NH119	Duncan River below B.B. Creek			
		50-38-17	117-02-50	1,330	24
140	08NH120	Moyie River above Negro Creek			
		49-25-20	115-56-28	240	23
141	08NJ013	Slocan River near Crescent Valley			
		49-27-38	117-33-52	3,320	62
142	08NJ014	Slocan River at Slocan City			
		49-46-08	117-28-23	1,660	28
143	08NJ061	Redfish Creek near Harrop			
		49-37-20	117-03-15	26.20	19
144	08NJ129	Fell Creek near Nelson			
		49-30-12	117-15-37	4.40	21
145	08NJ130	Anderson Creek near Nelson			
		49-30-05	117-15-35	9.07	21
146	08NK002	Elk River at Fernie			
		49-30-36	115-04-17	3,110	19
147	08NK012	Elk River at Stanley Park			
		49-18-42	115-03-04	3,520	25
148	08NK016	Elk River near Natal			
		49-51-58	114-52-06	1,870	32

149	08NL004	Ashnola River near Keremeos			
		49-12-33	119-59-23	1,050	39
150	08NL007	Similkameen River at Princeton			
		49-26-47	120-31-17	1,850	45
151	08NL036	Whipsaw Creek below Lamont Creek			
		49-22-09	120-34-11	185	23
152	08NL038	Similkameen River near Hedley			
		49-22-39	120-09-06	5,590	22
153	08NM015	Vaseux Creek Above Dutton Creek			
		49-15-44	119-28-27	255	21
154	08NM134	Camp Creek at Mouth near Thirsk			
		49-42-40	120-00-30	33.90	21
155	08NM142	Coldstream Creek above Municiple Intake			
		50-15-28	119-04-55	58.50	20
156	08NN002	Granby River at Grand Forks			
		49-02-39	118-26-19	2,050	24
157	08NN012	Kettle River near Laurier			
		48-59-04	118-12-55	9,840	58
158	08NN013	Kettle River near Ferry			
		48-58-53	118-45-55	5,750	58
159	08NN015	West Kettle River near McCulloch			
		49-42-15	119-05-31	230	22
160	08NN019	Trapping Creek near the Mouth			
		49-33-52	119-03-08	144	22
161	08NP001	Flathead River at Flathead			
		49-00-02	114-28-35	1,110	36
162	08OA002	Yakoun River near Port Clements			
		53-36-50	132-12-35	477	21
163	08PA001	Skagit River near Hope			
		49-02-50	121-05-45	907	25
164	09AA006	Atlin River near Atlin			
		59-35-57	133-48-48	6,810	37
165	09AA010	Lindeman Creek near Bennett			
		59-50-16	135-00-50	240	30
166	09AA013	Tutshi River at Outlet of Tutshi Lake			
		59-56-48	134-19-29	829	23
167	09AA014	Fantail River at Outlet of Fantail Lake			
		59-35-40	134-23-26	717	24
168	09AA015	Wann River near Atlin			
		59-25-55	134-12-20	269	23
169	09AE003	Swift River near Swift River			
		59-55-50	131-46-04	3,320	27
170	10AA001	Liard River at Upper Crossing			
		60-03-00	128-54-00	33,400	27

171	10AC002	Dease River at McDame			
		59-11-20	129-12-44	6,940	25
172	10AC003	Dease River at Outlet of Dease Lake			
		58-48-50	130-05-00	1,520	24
173	10AC004	Blue River near the Mouth			
		59-45-30	129-07-40	1,700	23
174	10AC005	Cottonwood River above Bass Creek			
		59-07-08	129-49-23	888	24
175	10AD001	Hyland River near Lower Post			
		59-57-03	128-09-03	9,450	27
176	10BA001	Turnagain River above Sandpile Creek			
		59-03-52	127-50-41	6,580	19
177	10BB001	Kechika River at the Mouth			
		59-36-55	127-18-37	22,700	21
178	10BC001	Coal River at the Mouth			
		59-41-29	126-57-02	9,190	24
179	10BE001	Liard River at Lower Crossing			
		59-24-45	126-05-50	104,000	31
180	10BE004	Toad River above Nonda Creek			
		58-51-20	125-22-50	2,570	24
181	10CB001	Sikanni Chief River near Fort Nelson			
		57-14-03	122-41-39	2,160	29
182	10CD001	Muskwa River near Fort Nelson			
		58-47-18	122-39-33	20,300	29

APPENDIX B
PARAMETER ESTIMATES FOR PARENT DISTRIBUTION

Station Identifier	First Component Mean	First Component Standard Deviation	Mixing Proportion	Second Component Mean	Second Component Standard Deviation
07EA001	-5.58509	0.41123	0.42323	-3.99613	0.92150
07EA002	-5.68087	0.48399	0.41529	-3.91871	0.96804
07ED001	-5.65136	0.36112	0.45942	-4.35923	1.00676
07EE007	-4.91979	0.49264	0.41802	-3.49898	0.92683
07FA001	-6.53336	0.53982	0.40742	-4.85883	1.02305
07FB001	-5.63346	0.51350	0.41756	-4.15855	0.97986
07FC001	-9.36006	1.07985	0.38356	-6.33483	1.68478
07FD001	-8.13064	0.77699	0.39198	-6.28421	1.46855
08CB001	-5.94762	0.47330	0.41257	-4.08546	0.97800
08CD001	-6.13571	0.43789	0.43130	-4.65117	1.01151
08CE001	-5.96719	0.43665	0.41416	-4.20159	0.93511
08CG001	-4.77221	0.53113	0.39715	-2.95812	0.91103
08CG003	-5.68122	0.43336	0.40981	-4.20637	0.84783
08CG004	-5.17725	0.52423	0.40393	-3.18552	0.99132
08DA005	-4.97400	0.65636	0.39360	-2.72621	1.10819
08DB001	-4.86673	0.60697	0.37284	-3.12638	0.88524
08DC006	-4.53329	0.61535	0.38910	-2.61485	0.97953
08DD001	-4.31790	0.54930	0.38650	-2.58889	0.87493
08EB003	-5.38216	0.49163	0.40152	-3.70473	0.89820
08EB004	-5.41440	0.60076	0.37856	-3.68828	0.91307
08EC001	-5.80126	0.38924	0.39938	-4.83917	0.68089
08ED002	-4.46167	0.44267	0.38806	-3.10509	0.71455
08EE004	-5.29001	0.52735	0.37915	-3.92045	0.78080
08EE008	-6.35795	0.66572	0.38438	-4.36553	1.05896
08EF001	-5.31103	0.52821	0.38542	-3.76482	0.84893
08EF005	-4.81553	0.50942	0.39387	-3.26783	0.84404
08EG011	-4.10510	0.60946	0.35388	-2.72718	0.81424
08EG012	-3.73947	0.67498	0.34118	-2.12925	0.86772
08FA002	-3.53723	0.43760	0.37152	-2.34439	0.63415
08FB002	-4.82202	0.44870	0.39987	-3.36930	0.75548
08FB007	-5.10709	0.45071	0.40458	-3.57205	0.80011

08FF001	-3.86103	0.55360	0.34846	-2.69281	0.74878
08FF002	-4.04604	0.61878	0.34358	-2.81114	0.89499
08FF003	-3.70292	0.56936	0.37228	-2.33867	0.82257
08GA024	-4.08187	0.49154	0.39789	-2.56636	0.83614
08GD004	-4.42166	0.44550	0.41608	-2.95310	0.84205
08HA001	-5.35754	1.02527	0.32460	-3.14041	1.22975
08HA003	-5.66095	0.85822	0.36359	-3.28725	1.23589
08HA010	-4.60202	0.85760	0.35611	-2.66084	1.13639
08HA016	-6.01423	0.69974	0.40493	-3.78488	1.35044
08HB004	-4.08127	0.65492	0.33339	-2.73397	0.81546
08HB014	-4.16954	0.84313	0.36229	-2.32506	1.17105
08HB024	-4.36211	0.82825	0.32885	-2.87489	1.06945
08HB029	-4.32112	0.65477	0.33503	-3.00729	0.79231
08HD001	-3.68200	0.42089	0.36879	-2.70090	0.63448
08HE006	-2.94211	0.41085	0.40924	-1.94998	0.79367
08JA002	-4.85516	0.43446	0.39459	-3.68353	0.69547
08JB002	-6.18511	0.38368	0.42658	-5.03112	0.77933
08JE001	-5.54813	0.34753	0.40453	-4.57653	0.59451
08KA001	-5.21438	0.49710	0.42618	-3.29654	1.06806
08KA004	-4.99627	0.44623	0.39756	-3.52395	0.78600
08KA005	-5.09807	0.44668	0.42315	-3.45656	0.92233
08KA007	-5.37370	0.45587	0.43057	-3.53455	1.03617
08KA008	-5.53050	0.52210	0.42435	-3.45630	1.14780
08KA009	-5.05406	0.47963	0.42435	-3.40196	1.00247
08KB001	-4.97940	0.45197	0.38929	-3.55555	0.74880
08KB003	-4.57094	0.56650	0.37140	-2.95227	0.82305
08KC001	-6.44277	0.39726	0.45543	-5.09822	1.06547
08KD001	-4.90786	0.42162	0.40138	-3.79167	0.71963
08KD003	-5.46184	0.41926	0.42514	-4.11379	0.86001
08KE009	-5.95508	0.49563	0.43150	-4.37480	1.04695
08KF001	-7.98199	0.48123	0.43195	-6.66719	1.08041
08KG001	-6.90417	0.29135	0.45853	-5.83841	0.81598
08KH001	-4.91148	0.43213	0.39550	-3.70580	0.70656
08KH003	-4.89366	0.48486	0.39180	-3.45064	0.80176
08KH006	-4.97673	0.43377	0.39656	-3.76107	0.70752
08KH010	-5.12424	0.46755	0.42165	-3.66106	0.92567
08KH014	-4.38613	0.45154	0.40741	-2.86433	0.81318
08KH019	-6.68891	0.53445	0.42630	-5.17091	1.06252
08LA001	-5.21565	0.45335	0.41334	-3.72074	0.86100
08LA004	-4.68159	0.42683	0.42334	-3.40383	0.85524
08LA007	-4.58657	0.44421	0.41458	-2.96746	0.88774
08LA008	-6.30647	0.52411	0.41959	-4.95803	0.97857
08LA013	-4.56399	0.47611	0.41097	-2.88061	0.88965
08LB020	-5.83963	0.39820	0.45747	-4.41786	1.06636

08LB022	-5.22593	0.45409	0.40955	-3.70431	0.85336
08LB047	-4.85776	0.48122	0.40788	-3.30029	0.86933
08LB064	-5.21080	0.44736	0.41182	-3.71706	0.84459
08LB069	-5.53073	0.43423	0.44729	-4.06827	1.06437
08LD001	-4.96463	0.46026	0.38870	-3.69886	0.82740
08LE024	-4.42519	0.44132	0.41756	-3.10228	0.84293
08LE027	-4.57703	0.48870	0.41577	-3.07257	0.92473
08LE031	-5.01212	0.35086	0.43234	-3.90743	0.73657
08LF022	-5.42944	0.37907	0.42308	-4.16434	0.76345
08LG048	-5.59752	0.52768	0.43538	-3.97836	1.13916
08LG056	-7.92842	0.50582	0.42725	-6.54305	1.06585
08MA001	-5.70712	0.40918	0.42356	-4.23589	0.82383
08MA002	-5.20123	0.44704	0.40861	-3.80017	0.78795
08ME004	-5.01903	0.57229	0.42065	-3.03557	1.09594
08MF003	-4.19628	0.44109	0.40328	-3.03815	0.76134
08MF062	-4.68238	0.43458	0.43645	-3.29770	0.96346
08MG003	-4.26200	0.55056	0.37506	-2.78231	0.80790
08MG004	-4.26954	0.45214	0.39437	-3.05818	0.74829
08MG005	-4.16867	0.43610	0.41166	-2.71006	0.81285
08MG006	-4.09339	0.53682	0.38210	-2.69840	0.87930
08MG007	-4.15332	0.50660	0.38486	-2.57547	0.79535
08MG013	-3.62873	0.37022	0.37538	-2.74551	0.56632
08MH001	-3.66750	0.33105	0.40330	-2.78027	0.59356
08MH006	-4.13349	0.70281	0.35596	-2.72943	1.02104
08MH016	-3.65564	0.35975	0.39726	-2.73292	0.61195
08MH029	-4.94953	0.49013	0.36823	-3.80063	0.81658
08MH056	-3.72620	0.39334	0.39430	-2.70569	0.67934
08MH076	-4.74663	0.81000	0.34806	-2.98647	1.05663
08MH103	-3.67787	0.34366	0.40467	-2.77393	0.61150
08MH104	-4.90602	0.31498	0.43944	-3.65525	0.86376
08MH105	-5.64498	0.56482	0.42200	-3.73126	1.19174
08NA002	-5.43513	0.34578	0.44807	-3.99614	0.88271
08NA024	-5.36261	0.15069	0.44645	-4.81808	0.41298
08NA045	-5.39040	0.25876	0.47091	-4.32638	0.81754
08ND006	-4.85880	0.40919	0.42946	-3.30834	0.89114
08ND009	-4.59855	0.48426	0.41346	-2.98607	0.90519
08ND012	-4.73015	0.44367	0.42372	-3.08896	0.93559
08ND013	-4.65599	0.44376	0.42643	-2.99159	0.93880
08ND014	-4.45488	0.52284	0.41770	-2.73283	1.00998
08NE001	-4.37281	0.45007	0.41867	-2.79547	0.88403
08NE006	-4.61222	0.40052	0.45142	-3.21779	1.02209
08NE008	-4.49166	0.31020	0.45119	-3.46399	0.76848
08NE039	-6.14432	0.48063	0.45447	-4.28071	1.28746
08NE074	-5.12562	0.40330	0.45432	-3.68613	1.04792

08NE077	-5.42956	0.38317	0.46681	-4.00550	1.12786
08NE087	-6.13008	0.39532	0.45972	-4.62905	1.10819
08NF002	-5.46092	0.36587	0.44180	-4.05470	0.91333
08NG005	-5.48371	0.36794	0.44155	-4.14310	0.90498
08NG012	-5.33720	0.40177	0.45732	-3.88607	1.08330
08NG042	-5.42180	0.37077	0.44520	-4.15938	0.89697
08NG046	-5.13787	0.41486	0.44996	-3.63538	1.06435
08NG051	-5.47161	0.35630	0.46283	-4.13439	1.03189
08NG053	-5.48369	0.35539	0.44456	-4.07621	0.92545
08NG065	-5.48097	0.36697	0.44479	-4.13295	0.92721
08NH001	-4.66513	0.44004	0.42168	-3.05812	0.89981
08NH005	-4.89401	0.41651	0.44115	-3.49136	0.96612
08NH006	-6.18005	0.48725	0.45351	-4.48016	1.23073
08NH007	-4.54411	0.37612	0.44048	-3.21897	0.87486
08NH032	-5.53186	0.46032	0.45221	-3.95016	1.17688
08NH034	-6.24815	0.50956	0.45886	-4.59561	1.29139
08NH066	-4.64921	0.37711	0.43771	-3.29045	0.86685
08NH084	-5.15458	0.34964	0.46912	-3.85044	1.03467
08NH115	-5.67128	0.29620	0.46663	-4.59263	0.87294
08NH119	-4.76848	0.47400	0.42563	-2.99522	0.99936
08NH120	-6.02858	0.55805	0.45341	-4.21784	1.38899
08NJ013	-4.80630	0.37121	0.44695	-3.56715	0.88365
08NJ014	-4.51356	0.33121	0.45421	-3.34106	0.84887
08NJ061	-5.06220	0.43604	0.46044	-3.58726	1.17081
08NJ129	-5.78107	0.45322	0.44560	-4.29943	1.08574
08NJ130	-5.98584	0.42260	0.44730	-4.50029	1.07302
08NK002	-5.35623	0.35903	0.44060	-4.15658	0.86289
08NK012	-5.25357	0.39875	0.44391	-3.99484	0.92881
08NK016	-5.61958	0.38495	0.44423	-4.23377	0.96451
08NL004	-6.51213	0.38050	0.46954	-5.04763	1.18520
08NL007	-6.00152	0.43535	0.45664	-4.49389	1.16849
08NL036	-6.86764	0.40961	0.46465	-5.41544	1.18582
08NL038	-6.36384	0.40947	0.46265	-4.87904	1.14967
08NM015	-7.11464	0.41804	0.47833	-5.58626	1.36532
08NM134	-6.71062	0.34026	0.46937	-5.54286	1.04179
08NM142	-7.19594	0.55205	0.43217	-5.63897	1.15504
08NN002	-6.37838	0.59478	0.43711	-4.36968	1.29864
08NN012	-6.64464	0.50019	0.45060	-4.91874	1.22297
08NN013	-6.90534	0.52583	0.45220	-5.09363	1.30670
08NN015	-6.43532	0.58018	0.44497	-4.52481	1.37587
08NN019	-6.67859	0.55337	0.44735	-4.83673	1.32783
08NP001	-5.26644	0.38800	0.46554	-3.84973	1.11824
08OA002	-4.08020	0.65565	0.35182	-2.75708	0.96847
08PA001	-4.71184	0.46366	0.41480	-3.42993	0.86150

09AA006	-5.15099	0.32941	0.40460	-4.12317	0.57035
09AA010	-5.47468	0.60863	0.41426	-3.19760	1.19486
09AA013	-5.30477	0.39918	0.41755	-3.81113	0.82221
09AA014	-5.88749	0.58779	0.42690	-3.53635	1.25753
09AA015	-5.45170	0.43296	0.44550	-3.61129	1.07966
09AE003	-5.60970	0.40773	0.41363	-4.16720	0.83413
10AA001	-5.88531	0.41008	0.41190	-4.38479	0.84038
10AC002	-5.72054	0.41869	0.41518	-4.14357	0.89403
10AC003	-5.87253	0.40976	0.42233	-4.54267	0.86112
10AC004	-5.90956	0.44152	0.40453	-4.47856	0.87152
10AC005	-5.58910	0.46913	0.41680	-3.85142	0.99834
10AD001	-5.92080	0.45225	0.41547	-4.20397	0.96333
10BA001	-5.96969	0.44959	0.41162	-4.24210	0.92943
10BB001	-5.91213	0.38029	0.42049	-4.40147	0.83719
10BC001	-5.86377	0.34295	0.43370	-4.46281	0.87073
10BE001	-5.93025	0.39629	0.41378	-4.40279	0.83415
10BE004	-5.57883	0.39315	0.43232	-3.98207	0.92758
10CB001	-5.99590	0.45783	0.41139	-4.45186	0.99672
10CD001	-6.72220	0.62333	0.39587	-4.59470	1.10659

APPENDIX C
LISTING OF TEST GAGING STATIONS

Station Identifier	Station Name / Latitude	Longitude	Basin Area	Record
08GA054	Mamquam River above Mashiter Creek			
	49-43-44"N	123-06-20"W	334 km^2	18 years
08FB006	Atnarko River near the Mouth			
	52-21-39	126-00-10	2,430	18
08KG001	West Road River near Cinema			
	53-18-40	122-53-15	12,400	18
08NK019	Grave Creek at the Mouth			
	49-50-36	114-51-36	619	18

REFERENCE LIST

Acreman, M.C., 1985, Predicting the Mean Annual Flood from Basin Characteristics in Scotland, Hydrological Sciences Journal, 30(1), pp. 37-50.

Aldenderfer, M.S., & R.K. Blasfield, 1984, Cluster Analysis, Sage University Paper Series on Quantitative Applications in the Social Sciences, series no. 07-044, Sage Publications, Inc., Beverly Hills.

Amedeo, D., & R.G. Golledge, 1975, An Introduction to Scientific Reasoning in Geography, John Wiley & Sons, Inc., New York.

Ashmore, P.E., & T.J. Day, 1988, Effective Discharge for Suspended Sediment Transport in Streams of the Saskatchewan River Basin, Water Resources Research, 24(6), pp. 864-870.

Bahadur, R.R., 1966, A Note on Quantiles in Large Samples, The Annals of Mathematical Statistics, 37, pp. 577-580.

Baker, V., 1985, Paleoflood Hydrology of Extraordinary Flood Events, U.S.-China Bilateral Symposium on the Analysis of Extraordinary Flood Events, Oct. 21-23, Nanjing, China.

Balkema, A.A., & L. de Haan, 1978a, Limit Distributions for Order Statistics. I, Theory of Probability and Its Applications, 23(1), pp. 77-92.

Balkema, A.A., & L. De Haan, 1978b, Limit Distributions for Order Statistics. II, Theory of Probability and Its Applications, 23(2), pp. 341-358.

Berman, S.M., 1962, Limiting Distribution of the Maximum Term in Sequences of Dependent Random Variables, The Annals of Mathematical Statistics, 33(3), pp. 894-908.

Box, G.E.P., 1976, Science and Statistics, Journal of the American Statistical Association, 71(356), pp. 791-799.

Bras, R.L. & I. Rodriguez-Iturbe, 1985, Random Functions and Hydrology, Addison-Wesley Pub. Co., Reading, MA.

Chang, M., & D.G. Boyer, 1977, Estimates of Low-Flows Using Watershed and Climatic Parameters, Water Resources Research, 13(6), pp. 997-1001.

Chibisov, D.M., 1964, On Limit Distributions for Order Statistics, Theory of Probability and its Applications, 9(1), pp. 142-148.

Chow, V.T., 1954, The Log-Probability Law and its Engineering Applications, Proceedings of the American Society of Civil Engineers, 80(536), pp. 1-25.

Chow, V.T., ed., 1964, Handbook of Applied Hydrology, McGraw-Hill Book Company, New York.

Cunnane, C., 1985, Factors Affecting Choice of Distribution for Flood Series, Hydrological Sciences Journal, 30(1), pp. 25-36.

Dalrymple, T., 1960, Manual of Hydrology: Part 3. Flood-Flow Techniques, Flood Frequency Analyses, U.S. Geological Survey Water Supply Paper 1543-A, U.S.G.S., Washington.

David, H.A., 1981, Order Statistics, second edition. John Wiley & Sons, Inc., New York.

Dempster, A.P., N.M. Liard, & D.B. Rubin, 1977, Maximum Likelihood from Incomplete Data via the EM Algorithm, Journal of the Royal Statistical Society, Series B, 39(1), pp. 1-22.

Department of Energy, Mines and Resources, Canada, 1973, The National Atlas of Canada, Department of Energy, Mines and Resources, Canada, Ottawa.

Dooge, J.C.I., 1986, Looking for Hydrologic Laws, Water Resources Research, 22(9), supplement, pp. 46S-58S.

Dracup, J.A., K.S. Lee, & E.G. Paulson, 1980a, On the Statistical Characteristics of Drought Events, Water Resources Research, 16(2), pp. 289-296.

Dracup, J.A., K.S. Lee, & E.G. Paulson, 1980b, On the Definition of Droughts, Water Resources Research, 16(2), pp. 297-302.

Environment Canada, 1980, Supplying Hydrometric and Sediment Data to Users, second edition, Environment Canada, Ottawa.

Environment Canada, 1982, Canadian Climate Normals: 1951-1980, Temperature and Precipitation, British Columbia, Environment Canada, Ottawa.

Environment Canada, 1988, Historical Streamflow Summary: British Columbia, Environment Canada, Ottawa.

Farley, A.L., 1979, Atlas of British Columbia, University of British Columbia Press, Vancouver.

Fiorentino, M., P. Versace & F. Rossi, 1985, Regional Flood Frequency Estimation Using the Two-Component Extreme Value Distribution, Hydrological Sciences Journal, 30(1), pp. 51-64.

Fisher, R.A., & L.H.C. Tippett, 1928, Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample, Proceedings of the Cambridge Philosophical Society, 24, pp. 180-190.

Foster, H.A., 1924, Theoretical Frequency Curves and Their Application to Engineering Problems, American Society of Civil Engineers, Transactions, 87(1532), pp. 142-203.

Foster, H.A., 1934, Duration Curves, American Society of Civil Engineers, Transactions, 99(1885), pp. 1213-1267.

Fraser River Board, 1958, Preliminary Report on Flood Control and Hydro-Electric Power in the Fraser River Basin, Fraser River Board, Victoria.

Freeze, R.A., 1975, A Stochastic Conceptual Analysis of One-Dimensional Groundwater Flow in Nonuniform Homogeneous Media, Water Resources Research, 11(5), pp. 725-741.

Galambos, J., 1978, The Asymptotic Theory of Extreme Order Statistics, John Wiley & Sons, Inc., New York.

Galambos, J., 1984, Order Statistics, pp. 359-382. Handbook of Statistics, Vol. 4: Nonparametric Methods, P.R. Krishnaiah & P.K. Sen eds., Elsevier Science Publishers B.V., Amsterdam.

Gambolati, G., G. Volpi, 1979, A Conceptual Deterministic Analysis of the Kriging Technique in Hydrology, Water Resources Research, 15(3), pp. 625-629.

Gibbons, J.D., 1985, Nonparametric Statistical Inference, Marcel Dekker, Inc., New York.

Golden Software, Inc., 1989, SURFER Reference Manual, Version 4, Golden Software, Inc., Golden, CO.

Gottschalk, L., 1985, Hydrological Regionalization of Sweden, Hydrological Sciences Journal, 30(1), pp. 65-84.

Gregory, K.J., & D.E. Walling, 1973, Drainage Basin Form and Process: A Geomorphological Approach, Edward Arnold Pub. Ltd., London.

Gries, N.P., 1983, Flood Frequency Analysis: A Review of 1979- 1982, Reviews of Geophysics and Space Physics, 21(3), pp. 699-706.

Gumbel, E.J., 1941, The Return Period of Flood Flows, Annals of Mathematical Statistics, 12, pp. 163-190.

Gumbel, E.J., 1954a, Statistical Theory of Droughts, Proceedings of the American Society of Civil Engineers, 80(439), p. 19.

Gumbel, E.J., 1954b, Statistical Theory of Extreme Values and Some Practical Applications, National Bureau of Standards, Washington.

Gumbel, E.J., 1958, Statistics of Extremes, Columbia University Press, New York.

Gupta, V.K., L. Duckstein & R.W. Peebles, 1976, On the Joint Distribution of the Largest Flood and Its Time of Occurrence, Water Resources Research, 12(2), pp. 295-304.

Haan, C.T., 1977, Statistical Methods in Hydrology, The Iowa State University Press, Ames.

Hazen, A., 1930, Flood Flows, John Wiley & Sons, New York.

Hirschboeck, K.K., 1985, Hydroclimatology of Flow Events in the Gila River Basin Central and Southern Arizona, Ph.D. dissertation, unpublished, University of Arizona.

Hoeffding, W., 1948, A Class of Statistics with Asymptotically Normal Distribution, The Annals of Mathematical Statistics, 19, pp. 293-325.

Hosking, J.R.M., & J.R. Wallis, 1986, Paleoflood Hydrology and Flood Frequency Analysis, Water Resources Research, 24(4), pp. 588-600.

Hosking, J.R.M., & J.R. Wallis, 1988, The Effect of Intersite Dependence on Regional Flood Frequency Analysis, Water Resources research, 24(4), pp. 588-600.

Hosking, J.R.M., J.R. Wallis, & E.F. Wood, 1985, Estimation of the Generalized Extreme-Value Distribution by the Method of Probability-Weighted Moments, Technometrics, 27(3), pp. 251-261.

Houghton, J.C., 1978, Birth of a Parent. The Wakeby Distribution for Modelling Flood Flows, Water Resources Research, 14(6), pp. 1111-1115.

Jarvis, C.S., 1936, Floods in the United States, Magnitude and Frequency, U.S. Geological Survey Water Supply Paper, No. 771.

Jenkinson, A.F., 1954, The Frequency Distribution of the Annual Maximum (or Minimum) Values of Meteorological Elements, Quarterly Journal of the Royal Meteorological Society, 87, pp. 158-171.

Jenkinson, A.F., 1969, Estimation of Maximum Floods, pp. 183-257, World Meteorological Office Technical Note 98, W.M.O., Geneva.

Johnston, R.J., 1978, Multivariate Statistical Analysis in Geography: A Primer on The General Linear Model, Longman Group Ltd., New York.

Journel, A.G., & Ch. J. Huijbregts, 1978, Mining Geostatistics, Academic Press Inc., Orlando.

Kendall, M., & A. Stuart, 1977, The Advanced Theory of Statistics, Vol. 1: Distribution Theory, fourth edition. Macmillan Pub. Co., Inc., New York.

Kinnison, R.R., 1985, Applied Extreme Value Statistics, Battelle Press, Columbus, OH.

Kite, G.W., 1977, Frequency and Risk Analyses in Hydrology, Water Resources Publications, Littleton, CO.

Klemeš, V., 1983, Conceptualization and Scale In Hydrology, Journal of Hydrology, 65, pp. 1-23.

Klemeš, V., 1986, Dilettantism in Hydrology: Transition or Destiny?, Water Resources Research, 22(9), pp. 177S-188S.

Kreuder, W.L., 1979, Development of Hydrometric Networks in British Columbia, Water Resources Research, 15(6), pp. 1677-1686.

Lamberti, P., & S. Pilati, 1985, Probability Distributions of Annual Maxima of Seasonal Hydrological Variables, Hydrological Sciences Journal, 30(1), pp. 111-136.

Leadbetter, M.R., 1978, Extreme Value Theory Under Weak Mixing Conditions, pp. 46-110. Studies in Mathematics, Vol. 18: Studies in Probability Theory, M.R. Rosenbatt Ed., The Mathematical Association of America.

Leadbetter, M.R., G. Lindgren & H. Rootzen, 1983, Extremes and Related Properties of Random Sequences and Processes, Springer-Verlag Inc., New York.

LeBoutillier, D.W., & P.R. Waylen, 1988, Stochastic Analysis of Extreme Cold Spells, Journal of Climate and Applied Meteorology, 27(1), pp. 67-76.

Leith, R.M., 1975, Streamflow Regionalization in British Columbia, No. 1. Regression of Mean Annual Floods on Physiographic Parameters, Report Series No. 40, Inland Waters Directorate, Pacific Region, Water Resources Branch, Vancouver.

Leith, R.M., 1976, Streamflow Regionalization in British Columbia, No. 2. Regression of Mean Annual Flows on Physiographic Parameters, Report Series No. 46, Inland Waters Directorate, Pacific Region, Water Resources Branch, Vancouver.

Leith, R.M., 1978, Streamflow Regionalization in British Columbia, No. 4. Regression of Low-Flows on Physiographic Parameters, Report Series No. 57, Inland Waters Directorate, Pacific Region, Water Resources Branch, Vancouver.

Leopold, L.B., M.G. Wolman & J.P. Miller, 1964, Fluvial Processes in Geomorphology, W.H. Freeman & Company, San Francisco.

Lettenmaier, D.P., & K.W. Potter, 1985, Testing Flood Frequency Estimation Methods Using a Regional Flood Generation Model, Water Resources Research, 21(12), pp. 1903-1914.

Lettenmaier, D.P., J.R. Wallis & E.F. Wood, 1987, Effect of Regional Heterogeneity on Flood Frequency Estimation, Water Resources Research, 23(2), pp. 313-324.

Leytham, K.M., 1984, Maximum Likelihood Estimates for the Parameters of Mixture Distributions, Water Resources Research, 20(7), pp. 896-902.

Loynes, R.M., 1965, Extreme Values in Uniformly Mixing Stationary Stochastic Processes, The Annals of Mathematical Statistics, 36(3), pp. 993-999.

Matalas, N.C., 1963, Probability Distribution of Low-Flows, U.S. Geological Survey Professional Paper, 434-A.

Matalas, N.C., 1967, Mathematical Assessment of Synthetic Hydrology, Water Resources Research, 3(4), pp. 937-945.

Matalas, N.C., J.R. Slack & J.R. Wallis, 1975, Regional Skew in Search of a Parent, Water Resources Research, 11(6), pp. 815-826.

Matalas, N.C., & J.R. Wallis, 1973, Eureka! It Fits a Pearson Type III Distribution, Water Resources Research, 9(2), pp. 281-289.

Monger, J.W.H., & R.A. Price, 1979, Geodynamic Evolution of the Canadian Cordillera - Progress and Problems, Canadian Journal of Earth Sciences, 16(3), pp. 770-792.

Mood, A.M., F.A. Graybill & D.C. Boes, 1963, Introduction to the Theory of Statistics, third edition. McGraw-Hill Book Company, New York.

Mosley, M.P., 1981, Delimitation of New Zealand Hydrological Regions, Journal of Hydrology, 49 pp. 173-192.

National Research Council, 1988, Estimating Probabilities of Extreme Floods: Methods and Recommended Research, National Academy Press, Washington.

Natural Environment Research Council, 1975, Flood Studies Report, Vol. I: Hydrological Studies, Natural Environment Research Council, London.

Olea, R.A., 1974, Optimal Contour Mapping Using Universal Kriging, Journal of Geophysical Research, 79(5), pp. 695-702.

Pipes, A., M.C. Quick, & S.D. Russell, 1970, Simulating Snowmelt Hydrographs for the Fraser River System, Proceedings of the 38th Western Snow Conference, pp. 91-97.

Potter, K.W., 1987, Research on Flood Frequency Analysis: 1983-1986, Reviews of Geophysics, 25(2), pp. 113-188.

Richards, K., 1982, Rivers: Form and Processes in Alluvial Channels, Methuen & Co. Ltd., London.

Rossi, F., M. Fiorentino & P. Versace, 1984, Two-Component Extreme Value Distribution for Flood Frequency Analysis, Water Resources Research, 20(7), pp. 847-856.

Sampson, R.J., 1978, SURFACE II Graphics System, Kansas Geological Survey, Lawrence.

Sarhan, A.E., & B.G. Greenberg, eds., 1962, Contributions to Order Statistics, John Wiley & Sons, Inc., New York.

SAS Institute, Inc., 1988, SAS/STAT User's Guide, Release 6.03 Edition, SAS Institute, Inc., Cary, NC.

Searcy, J.C., 1959, Manual of Hydrology: Part 2. Low-Flow Techniques, Flow-Duration Curves, U.S. Geological Survey Water Supply Paper 1542-A, U.S.G.S., Washington.

Sim, C.H., 1987, A Mixed Gamma ARMA(1,1) Model for River Flow Time Series, Water Resources Research, 23(1), pp. 32-36.

Smith, D.M., 1975, Patterns in Human Geography: An Introduction to Numerical Methods, Crane, Russk & Co., Inc., New York.

Smith, J.A., 1987, Estimating the Upper Tail of Flood Frequency Distributions, Water Resources Research, 23(8), pp. 1657-1666.

Smith, J.A., 1989, Regional Flood Frequency Analysis Using Extreme Order Statistics of the Annual Peak Record, Water Resources Research, 25(2), pp. 311-317.

Smith, J.A., & A.F. Karr, 1986, Flood Frequency Analysis Using the Cox Regression Model, Water Resources Research, 22(6), pp. 890-896.

Smirnov, N.V., 1952, Limit Distributions for the Terms of a Variational Series, American Mathematical Society, Translation Number 67.

Smirnov, N.V., 1967, Some Remarks on Limit Laws for Order Statistics, Theory of Probability and its Applications, 12(2), pp. 337-339.

Takahashi, R., 1987, Normalizing Constants of a Distribution Which Belongs to the Domain of Attraction of the Gumbel Distribution, Statistics and Probability Letters, 5(3), pp. 197-200.

Unwin, D.J., 1975, Concepts and Techniques in Modern Geography No. 5: An Introduction to Trend Surface Analysis, Geo Abstracts, Norwich.

Water Resources Council, 1967, A Uniform Technique for Determining Flood Flow Frequencies, Bulletin No. 15, U.S. Water Resources Council, Washington.

Watson, J.D., 1934, Discussion of Duration Curves, American Society of Civil Engineers, Transactions, 99, pp. 1245-1246.

Watts, V., 1980, The Almost Sure Representation of Intermediate Order Statistics, Zeitschrift fur Wahrscheinlichkeitstheorie und verwandte Gebiete, 54(3), pp. 281-285.

Watts, V., H. Rootzen & M.R. Leadbetter, 1982, On Limiting Distributions of Intermediate Order Statistics from Stationary Sequences, The Annals of Probability, 10(3), pp. 653-662.

Waylen, P.R., 1985a, Stochastic Flood Analysis in a Region of Mixed Generating Processes, Transactions of the Institute of British Geographers, 10, pp. 95-108.

Waylen, P.R. 1985b, A Method of Predicting Daily Peak Flows in the High-Flow Season, Journal of Hydrology, 77, pp. 89-105.

Waylen, P.R. & C.N. Caviedes, 1986, El Nino and Flooding on the North Peruvian Littoral, Journal of Hydrology, 89, pp. 141-156.

Waylen, P.R. & M-k. Woo, 1982, Prediction of Annual Floods Generated by Mixed Processes, Water Resources Research, 18(4), pp. 1283-1286.

Waylen, P.R. & M-k. Woo, 1983, Annual Floods in Southwestern British Columbia, Canada, Journal of Hydrology, 62, pp. 95-105.

Waylen, P.R. & M-k. Woo, 1984, Regionalization and Prediction of Floods in the Fraser River Catchment, B.C., Water Resources Bulletin, 20(6), pp. 941-949.

Waylen, P.R., & M-k. Woo, 1987, Annual Low-Flows Generated by Mixed Processes, Hydrological Sciences Journal, 32(3), pp. 371-383.

Wiltshire, S.E., 1985, Grouping Basins for Regional Flood Frequency Analysis, Hydrological Sciences Journal, 30(1), pp. 151-160.

Wiltshire, S.E., 1986a, Regional Flood Frequency Analysis I: Homogeneity Statistics, Hydrological Sciences Journal, 31(3), pp. 321-334.

Wiltshire, S.E, 1986b, Regional Flood Frequency Analysis II: Multivariate Classification of drainage Basins in Britain, Hydrological Sciences Journal, 31(3), pp. 335-346.

Wolman, M.G., & J.P. Miller, 1960, Magnitude and Frequency of Forces in Geomorphic Processes, Journal of Geology, 68, pp. 54-74.

Woo, M-k., & P.R. Waylen, 1984, Areal Prediction of Annual Floods Generated by Two Distinct Processes, Hydrological Sciences Journal, 29(1), pp. 75-88.

Woolhiser, D.A., & J. Roldan, 1986, Seasonal and Regional Variability of Parameters for Stochastic Daily Precipitation Models: South Dakota, U.S.A., Water Resources Research, 22(6), pp. 965-978.

World Meteorological Organization, 1981, Guide to Hydrological Practices Volume I; Data Acquisition and Processing, fourth edition, World Meteorological Organization, Geneva.

Wu, Chuan-yi, 1966, The Type of Limit Distributions for some Terms of Variational Series, Scientia Sinica, 15(6), pp. 749-762.

Yevjevich, V., & J.T.B. Obeysekera, 1985, Effects of Incorrectly Removed Periodicity in Parameters on Stochastic dependence, Water Resources Research, 21(5), pp. 685-690.

Zelenhasic, E., 1970, Theoretical Probability Distributions for Flood Peaks, Hydrology Paper No. 42, Colorado State University, Fort Collins, CO.

Zelenhasic, E., & A. Salvi, 1987, A Method of Streamflow Drought Analysis, Water Resources Research, 23(1), pp. 156-168.

BIOGRAPHICAL SKETCH

David W. LeBoutillier was born and raised in Saskatoon, Saskatchewan. He came to the University of Florida after earning both a bachelor's and a master's degree in geography from the University of Saskatchewan. While at the University of Florida, David has had numerous opportunities to travel to many professional conferences and worked for International Business Machines Corporation as a vector assistant. David is looking forward to his future career as an instructor and researcher at Indiana State University in the department of Geography and Geology.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Peter R. Waylen, Chairman
Associate Professor of Geography

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

César N. Caviedes
Professor of Geography

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

James A. Henry
Associate Professor of Geography

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Wayne C. Huber
Professor of Environmental Engineering Sciences

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Rocco Ballerini
Associate Professor of Statistics

This dissertation was submitted to the Graduate Faculty of the Department of Geography in the College of Liberal Arts and Sciences and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

August, 1990

_____

Dean, Graduate School

UNIVERSITY OF FLORIDA
3 1262 08285 298 8